STOCHASTIC ANALYSIS OF SINGLE QUEUE SINGLE SERVER VERSUS SINGLE QUEUE MULTIPLE SERVERS MODELS: A CASE STUDY OF POST BANK AND KENYA COMMERCIAL BANK

BY

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DECLARATION

I, Nicholus Sewe, duly declare that this project is my own original work and it has not been presented to any university for degree or any other award.

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DEDICATION

I dedicate this project to my family. Indeed, we had the same aspiration to work towards its logical conclusion.

ABSTRACT

Banks play significant roles in a country's economy. For this reason many studies have been done on the management and general organization of banks. One such area is on queue management. It is common practice to see long queues of customers waiting to be served within the banking halls. Customers arrive at banking facilities randomly. Moreover, service time is also a random phenomenon. Currently, many institutions are moving away from single queue single server model to single queue-multiple servers model, Presumably, because the waiting time in the latter model is lower but is this always the case? In our study we compared single queue single server to single queue multiple server: A case study of Post Bank Kisumu and Kenya Commercial Bank Kisumu. In both models we have assumed that the arrival times follow a Poisson process while service times follow an exponential distribution. Our main parameter of interest is the waiting time.We have used $M/M/1$ and $M/M/r$ to study the two models and determine the preferable model for any specific situation. In our study we found that although the average waiting time in Post Bank is greater than that in the Kenya Commercial Bank, the equivalence of the KCB average waiting time to the Post Bank is higher. Further, the difference between the means in the waiting times in the two banks is significant at 5% significance level.

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Chapter 1

INTRODUCTION

1.1 Background information

Banks play significant roles in Kenyan economy. For this reason many studies have been done on the management and general organization of banks . It is common practice to see long queues of customers waiting to be served within the banks. Customers arrive at banking facilities randomly. Moreover, sevice time is also a random phenomena. A queue occurs where facilities are limited and cannot satisfy the demand made against them at a particular period. However, time in queues are formed when customers demanding services have to wait because their numbers exceed the number of servers available or the facility does not work efficiently. Some customers wait when the total number of customers waiting for the services exceeds the number of service facilities available, however, some service facilities stand idle when the total number of service facilities exceeds the number of customers requiring the service. Queues are associated with order although chaos usually erupt whenever customers try to jump queues. A queue is therefore a waiting line [3] alternatively it is a waiting line by two important elements for instance the population source of customer from which they can draw the service system [4]. The source from which the customers are generated could be finite(customers arrival is limited) or infinite (customer arrival not limited). Waiting line management is the greatest dilemma for managers seeking to improve the investment of their operations since customers do not tolerate waiting. Whenever a customer feels that he/she has waited too long at a station for service, he/she may either opt out prematurely or may come back later.

1.1.1 Effects of long queues in banking industry

Long queues not only inconvenience customers but also frustrate their daily lives. Often customers may be discouraged from pursuing valuable services by a sheer length of waiting line. At times waiting may make the customer delay or miss an important business transaction. If the arrival rates, queue system, queue discipline and the service rates are known, characteristic of queue in a steady state can be calculated. These include the average waiting time experienced by the customer, the average number of customers waiting to be served in the queue or the servers. [8] confirmed that even if the system can provide a service at a faster rate than arrival rate,waiting lines can still form if the arrival and the service rates are random.

1.1.2 Queuing systems

The queuing system in the banks' consist of the following:

Customer area

This is where the customer arrives and makes a decision whether to join the queue or not.

The queuing area

This is where the customers join the queue as they wait for the service.

Service area

This is where the system serves the customer according to different services that the automated teller machine provides. For instance, cash withdrawal, inquiring account balance and others.

1.2 Statement of the problem

Queues have become a common problem in many banks in Kenya more so during the end of the month when the withdrawals and deposits are at high rates. Long queues cause delay in service delivery as well as constraints. Therefore banks must strive to satisfy and provide better services by designing better strategies of managing lengthy queues. Currently, many institutions where queues form an integral part of service delivery, appear to be moving from single queue- single server model to single queue multiple servers model. However, there is hardly any mathematical model that directly compares the two systems with the aim of giving conditions in which any system is preferable. In our study we compared the waiting times for single queue-single server and single queue multiple servers models and determine conditions under which a specific model is preferable.

1.3 Objectives of the study

1.3.1 General objective

The general objective of this study is to compare waiting times in KCB and Post Bank and to analyze their queuing systems so as to understand the behavior of their underlying processes for informed decisions to be undertaken by the banks' managers.

1.3.2 Specific objectives

- To analyze the waiting time in single queue single-server model at Post Bank, Kisumu.

- To analyze the waiting time in single queue single-server model at KCB, Kisumu main branch.

- To compare the performance of the different systems in varying conditions.

1.4 Significance of the study

Waiting in a line is a daily phenomenon experienced by customers waiting to be served in banking halls, ATMs, hospitals supermarkets, post offices, railway stations and other public places. The efficiency of queuing system is one of the factors that influence customers' perception on quality of service.

Queuing is psychologically stressing to customers especially if she/he is doing nothing. For instance, in banking halls , the most common system comprises of a single queue and a multiple service stations (servers). Queues normally form because of less service capacities than the demand and therefore the queuing model seeks to find the optimum service rate rate and the optimal number of servers. Thus the service time needs to be improved to maintain the customers. Boredom due to queuing generates relentlessness,tension and anxiety.

This study wil be used to improve services at the bank's sections in order to increase quality of service by making queue management an active part of the bank's strategic plan. Moreover, the result of this study can act as a reference point in analyzing and improving the next system since the bank will now be able to estimate the number of customers going away each day without being served.

1.5 Justification of the study

Queuing system primarily involves the provision of services. These systems involves the arrival and departure of customers at service centers in search of efficient and quick services. Queuing system extend beyond waiting lines in the banks and in the banking halls and the usual phenomena of delay caused by busy servers. The systematic study of the queuing system may be useful in contributing towards other areas in the society such as:

- Analysis of reducing waiting time in the bank's section.
- Reduction of queues in the bank's section will attract more customers to join the bank's services.

1.6 Basic concepts

1.6.1 Poisson distribution

Waiting line model assumes that the number of arrivals occurring within the interval time t follows Poisson distribution with parameters λt which is also equal to the variance. The probability density function is thus given by:

$$P_n(t) = \begin{cases} \frac{\lambda t^n}{n!}\ell^{-\lambda t}, & \text{n} = 1, 2, 3 \cdots \\ 0, & otherwise \end{cases} \tag{1.1}$$

Where;

$$E(n) = \lambda t \tag{1.2}$$

$$Var(n) = \lambda t \tag{1.3}$$

The Poisson distribution is suitable for depicting random behavior of individual events that occur relatively and frequently within the time span t [21].

1.6.2 Exponential distribution

Normally used to depict the behavior of the random behavior of time interval between the occurrences of two consecutive events thus has a probability density function given by (**??**) below[21]. In most cases queuing situations, arrival of customers occur in totally random trend i.e the arrival or completion of service is not influenced by the length of time that has elapsed since the occurrence of the last event.

$$f(t) = \lambda e^{-\lambda t}, \lambda \ \geq \ 0 \tag{1.4}$$

$$E(t) = \frac{1}{\lambda} \tag{1.5}$$

$$Var(t) = \frac{1}{\lambda^2} \tag{1.6}$$

$$\begin{aligned} P\{t \leq T\} &= \int_0^T \lambda e^{-\lambda t} dt \\ &= 1 - e^{-\lambda t} \end{aligned} \tag{1.7}$$

1.6.3 Little's formula

Consider an arrival leaving the system after the service after spending an average waiting time (W) in the system . Still in the system are are an average of L customers who arrived at an average interval of ($\frac{1}{\lambda}$). In the steady state, the two average times must be equal hence

$$W = L(\frac{1}{\lambda}) \tag{1.8}$$

$$L = \lambda W \tag{1.9}$$

The most commonly used measures of performance in queuing situations are:
$L_s$is the expected number of customers in the system.
L_q is the expected number of customers in the queue.
W_s is the expected waiting time in the system.
W_q is the expected time in the queue.

r is the number of servers.

$$L = L_q + \frac{\lambda}{\mu} \quad (1.10)$$

$$W = W_q + \frac{1}{\mu} \quad (1.11)$$

Hence,

$$L_q = \lambda W_q \quad (1.12)$$

Time is an essential commodity in life and need not to be wasted in the queues. Analysis of waiting time in banks it is done using the developed models based on a simple Markovian model for data collection and analysis. It is collected during different sessions that are peak days and non-peak days. In addition, other sources of information for this study are the internet, Maseno research library, past research work articles journals and relevant literature.

1.7 Project assumptions

In this study we consider the dynamics of queues or waiting lines where we assume that :

- Customers arrivals at the service station is random and independent to each other.

- Upon arrival, customers wait in the queue until it is their turn to be served.

- Once the customer is served, he/she leaves the system.

- First come first serve basis is used.

- one queue multiple service stations is used.

- Arrivals are in accordance with the Poisson process.

- The average service rate is faster than the average arrival rate.

- Group arrival is treated as a single arrival.

- No balking, no reneging no jockeying and collusion

- Service times are exponentially distributed

1.7.1 Essential components of queuing phenomenon

Population source/arrival process

The population source serves as where arrivals are generated. Arrivals of the customers at the bank may be drawn from either a finite or infinite population. A finite population refers to a limited size of customer pool while infinite source is unlimited.

Queue discipline

Queue discipline is the sequence where customers are served on the basis of their arrival time i.e First Come First Serve(FCFS).Other queue characteristics include: Last Come First Serve(LCFS) and service in random order(SIRO). Customers may also be selected from the line based on the order of priority [19].

Service mechanism

This mechanism describes how customers are served. It includes of the number of servers and the duration of service time. The number of lines and servers determines the choice of service facility structures. The common service facility structures are: single-channel, single phase; single-channel, multiphase; multichannel, single phase and multi-channel, multiphase.

Departure or exit

Departure or service occur when a customer has been served. The two possible exit scenarios as mentioned by [15] are:

- The customer may return to the source population and immediately becomes a competing candidate for service.

- There may be a low probability of re-service.

Chapter 2

LITERATURE REVIEW

Waiting causes not only inconvenience but also frustrations to people's daily lives. Often, customers may be discouraged from pursuing valuable services by a sheer length of waiting line. At other times, waiting may also make the customer delay or miss an important event. The queues are formed when the demand for a service exceeds its supply.[12]

Erlang, a Danish engineer, who worked in Copenhagen wrote the first paper on queue theory in the field of telephony in 1909. The study identified that the number of telephone conversations and telephone holding time fit into Poisson distribution and are exponentially distributed repectively. Most of the results of study are in use to date and apply to many seemingly unrelated situations from serving customers at service counters to managing traffic congestion in a cosmopolitan city and from designing switching equipment for telecommunications to understanding Internet behavior.

A few academic research papers have been curried out in the area of queue management and banks in Kenya. Kithaka [1] wrote a thesis to asses the extent to which a queue management in financial institutions is applied in Kenya. He observed that the institutions with enhanced queue management system remain competitive in customer service initiative. This more so in banks which have superior branch networking as well as training of employees and improvement in devolution of decision making.

Safe Associate[5] re-enforced the three main characteristics that determine the suitability of a queue model as the arrival to the system, queue discipline and service facility. The size, pattern of the arrival and service time distribution give rise to the arrival characteristics and also demonstrated that a single server has a Poisson distribution of arrival while the service is exponentially distributed.

Vazsonyi[26] observed that queuing theory provides a good conceptual model of waiting line conditions because it gives one a general understanding of influence of such factors as arrival distribution, service distribution and number of servers on queue conditions. Little's theorem[10] describe the relationship between arrival and service ,cycle time and work in process i.e the number of customers in the queue. The theorem states that the expected number of customers (N) in steady state can be determined. Katz [2] concluded that long queues has negative impacts on customers evaluation on outlets' quality since it affects the customers' perception of the "punctuality" of a service and hence the customers ratings of the service providers' efficiency and reliability.

Hongna and Zhenwei(2010) stated that queuing in a bank is a common phenomenon as well as a naughty problem. They collected line data of banks, obtained distributions and parameters on customers' arrival and service time intervals. They also assessed two system queuing strategies by taking the average length of stay and time in a queue system [11]. Palm (1943) and Kchinchin (1960)[14] expressed tha the arrival processes of customers to a service facility will tend to Poisson process. Lariviere et al. [13] affirmed that the specification of inter-arrival times has exponential distribution and is accurate for many service system. Chase and Aquiliano(2006) outlined three factors used in queue system management as length of line, number of lines and queue discipline [27].

Davis [15] asserted that providing ever first service with ultimate goal of having zero customer waiting time has received managerial attention for several reasons. First, in the first world countries with high standards of living, time becomes more valuable commodity hence customers are less willing to wait for service. Secondly, there is a growing realization by the organizations that the way they treat their customers today significantly impacts whether they will remain loyal or not. Moreover, technology such as computers, internet among others have made the provision of services faster.

Researchers have argued that service waits can be controlled by two techniques: operations management or perception management[16]. The operation management deals with organization of how customers, queues and customers can be coordinated towards the goal of rendering efficient services at minimum cost possible. The act of waiting has significant impact on customers satisfaction.Moreover research has demonstrated that customer is not only affected by the waiting time but also customers' expectations of the causes of waiting[17]. Agner Krarup Erlang created models to describe the Copenhagen telephone exchange[19]. His ideas have been applied in telecommunications,traffic engineering, computing and design of factories,shops, offices and hospitals [9, 20].

Udayabhanu et al [24] established that congestion in queuing system has serious consequences such that it is never optimal to operate at 100% utilization levels. They developed an expression for the optimal utilization level for an M/D/1 queue and demonstrated its similarity to economic order quantity, EOQ, model of the inventory literature.

Chapter 3

METHODOLOGY

3.1 Measures of performance for queuing systems

- The distribution of arrival rate of customers.

- The distribution of service rate (length of stay) of customers in one server.

- System utilization factor.

- Inter-arrival time between customer $(n-1)$ and customer n.

- Total time spent in the queue by customers.

- Average number customers in the system.

- Average number of customers in the queue.

3.2 Suggested queuing models

We have used Birth and Death process Queuing models under steady state conditions to study the queuing systems at Kenya Commercial Bank and Post Bank, Kisumu.

3.2.1 Birth and death processes

For the M/M/1 Queuing model:

A queuing system based on birth and death process is the state $E(n)$ at time t if the number of customers is $n = 0, 1, 2, \cdots$ i.e $Z(t) = n$. A birth occurs when a customer arrives and a death occurs when a customer leaves the system after the service. Given the birth rates (λ_n) and death rate (μ_n) and assuming that $Z(t)$ be the population size at time t and let $P_{in}(t)$ be a transition probability that the number of customers in the system at time t is equal to n when at time t is equal to zero they are high, thus;

$$P_{in}(t) = f(Z(t) = n) \tag{3.1}$$

Birth process

The probability that a birth occurs (a customer arrives into the system) from a population of size (N) within the time interval $(\triangle t)$ is given by:

$$P(one\, birth) = \lambda_n(\triangle t) + o(\triangle t) \tag{3.2}$$

Where ;
$o(\Delta t)$ is the probability of more than one birth and $o(\Delta t)$ is such that $\frac{o(\Delta t)}{\Delta t}$ tends to zero as Δt tends to zero.

Death process

The probability that one death occurs in a population of size (N) i.e a customer leaves the system after service is given by:

$$P(one\, death) = \mu_n \triangle t + o(\triangle t) \tag{3.3}$$

Neither birth nor Death

This is the probability that the number of customers in the system at the initial time is the same as the number of customers in the system at time t.

$$1 - (\mu_n + \lambda_n) \triangle t + o(\triangle t) \tag{3.4}$$

Combining(3.2), (3.3)(3.4) we obtain

$$P_n(t + \triangle t) = [\lambda_{n-1} \triangle t + o \triangle t]P_{n-1}t + [(\mu_{n+1} \triangle t + o(\triangle t))P_{n+1}t] + [1 - (\lambda_n + \mu_n) \triangle t + o \triangle t]P_n(t), n \geq 1 \tag{3.5}$$

$$P_n(t) = \lambda_{n-1} \triangle P_{n-1}(t) + \mu_{n+1} \triangle P_{n+1}(t) + [1 - (\lambda_n + (\mu_n) \triangle tP_n(t)] \tag{3.6}$$

$$for\ n \geq 1$$

Differentiating (3.6)$with\ \ respect\ \ to\,t$

:

$$P_n^{;}(t) = \lambda_{n-1}P_{n-1}(t) + \mu_{n+1}P_{n+1}(t) + [1 - (\lambda_n + (\mu_n)P_n(t)] \tag{3.7}$$

At steady states

$$P_n^{'}(t) = 0 \tag{3.8}$$

Hence,

$$P_n^{'}(t) = \lambda_{n-1}P_{n-1}(t) + \mu_{n+1}P_{n+1}(t) - (\lambda_n + \mu_n)P_n(t)] = 0 \tag{3.9}$$

Thus:

$$\lambda_{n-1}P_{n-1}(t) + \mu_{n+1}P_{n+1}(t) = (\lambda_n + \mu_n)P_n(t) \tag{3.10}$$

Dropping t for simplicity we obtain the balance equation:

$$\lambda_{n-1}P_{n-1} + \mu_{n+1}P_{n+1} = (\lambda_n + \mu_n)P_n \tag{3.11}$$

If $n = 0$ the balance equation becomes

$$\lambda_0 P_0 = \mu_1 P_1 \tag{3.12}$$

for which

$P_1 = (\frac{\lambda_0}{\mu_1})P_0$ and $n = 1$ we have:

$$\lambda_0 P_0 + \mu_2 P_2 = (\lambda_1 + \mu_1)P_1 \tag{3.13}$$

Substituting $P_1 = (\frac{\lambda_0}{\mu_0})P_0$ into eqref for $n = 1$ we obtain

$$P_2 = (\frac{\lambda_1 \lambda_2}{\mu_2 \mu_1})P_0 \tag{3.14}$$

In general,

$$P_n = (\frac{\lambda_{n-1}\lambda_{n-2}.........\lambda_0}{\mu_n \mu_{n-1}..........\mu_1})P_0 \tag{3.15}$$

Letting $C_n = (\frac{\lambda_n \lambda_{n-1} \lambda_1}{\mu_n \mu_{n-1} \mu_1})$ and $C_n = 1 \ for \ n = 0$ we get the following state probabilities

$$P_n = c_n p_0 \ for \ n = 0, 1, 2.... \tag{3.16}$$

$$\tag{3.17}$$

The requirement that

$$\sum P_n = 1.$$

implies that $(\sum c_n)p_0 = \sum P_n = 1, r = 1$ the c_n factor for the birth and death process reduces to:

$$c_n = \frac{(\lambda)}{(\mu)} = \rho^n \ for \ n = 0, 1, 2...... \tag{3.18}$$

$\rho = \frac{\lambda}{\mu}$ being the traffic intensity or utilization factor

Therefore,

$p_n = \rho^n p_0 \ for \ n = 0, 1, 2$ where $p_0 = (\sum \rho^n)^{-1} = (\frac{1}{1-\rho})^{-1} = 1 - \rho$

Hence

$$P_0 = 1 - \rho \tag{3.19}$$

This is the probability that a server is free when the customer arrives.

Thus

$$P_n = (1 - \rho)\rho^n \ for \ n = 0, 1, 2........ \tag{3.20}$$

$$\tag{3.21}$$

The expected number of customers in the queue including those being served ,L, is given by:

$$E(N) = L = \sum nP_n = \sum n(1-\rho)\rho^n \tag{3.22}$$

Which solves to:

$$L = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda} \tag{3.23}$$

$$\tag{3.24}$$

and for L_q , the number of customers in the queue excluding those being served: $L_q = \sum(n-1)P_n$ since $r = 1$, $L - 1(1 - P_0) = \frac{\lambda^2}{\mu(\mu-\lambda)}$)
Hence:

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} \tag{3.25}$$

3.3 The M/M/r Queuing model:

In the M/M/r queuing model, customers line up in a single queue in front of r parallel servers. A customer, leading in the queue immediately goes to a server from which a customer has just exited. The stationary process for the queue with r servers is

$$\mu P_1 - \lambda P_0 = 0 \tag{3.26}$$

$$\lambda P_{n-1} + (n+1)\mu P_{n+1} - (\lambda + n\mu)P_n = 0, \qquad 1 \leq n < r \tag{3.27}$$

$$\lambda P_{n-1} + r\mu P_{n+1} - (\lambda + r\mu)P_n = 0, \qquad n \geq r \tag{3.28}$$

which summaries to

$$n\mu P_n = \lambda p_{r-1}, 1 \leq n < r \tag{3.29}$$

$$r\mu P_n = \lambda P_{n-r}, n \geq r \tag{3.30}$$

$$\tag{3.31}$$

By iteration starting with n=1 $P_1 = \frac{\lambda}{\mu} P_0 = \rho p_0$ Further subsequent iterations yield for $n \geq r$

$$P_n = \frac{\rho^n P_0}{n!} \tag{3.32}$$

$$for\ n \geq r \tag{3.33}$$

$$P_n = \frac{\rho^n}{r^{n-r} r!}$$

Since$\sum P_n = 1, P_0$ can be determined from $P_0[\sum_{n=0}^{r-1} \frac{\rho^n}{n!} + \frac{r^n}{r!} \sum_{n=r}^{\propto} (\frac{\rho}{r})^n] = 1$ In which the infinite geometric geometric series converges if $P < r$ this gives P_0 as

$$P_0 = \frac{1}{\sum_{n=0}^{r-1} [\frac{\rho^n}{n!} + \frac{\rho^r}{(r-\rho)(r-1)!}]} \tag{3.34}$$

If N is the random number of queue length n, then the expected queue length excluding those being served is:

$$E(N) = \sum_{n=r+1}^{\propto} (n-r) P_n \tag{3.35}$$

$$= (\frac{P_0 \rho^r}{r!} \sum_{n=r+1}^{\propto} (n-r) (\frac{\rho}{r})^{n-r} \tag{3.36}$$

Summing up the series yields

$$E(N) = \frac{P_0 \rho^{r+1}}{(r-1)!(r-p)^2} \tag{3.37}$$

This formula gives the expected length of queue for those waiting to be served.
Utilizing this model:

$$\rho = \frac{\lambda}{r\mu} \tag{3.38}$$

The mean length of queue for many queues many servers i.e r/M/M/1 model where customers arrive at different queues with no choice of queue to join is given by:

$$E(N) = \frac{r\rho^2}{1-\rho} \tag{3.39}$$

Where E(N) is in multiples of r servers for the M/M/1 model. For instance, for two such queues , the equation is given by:

$$E(N) = \frac{2\rho^2}{1-\rho} \tag{3.40}$$

3.4 Data collection

3.4.1 Systematic sampling technique

This is type of probability sampling method in which the sample size is selected according to a random starting point with a fixed periodic interval. This interval is called sampling interval. In our study, we intend to use systematic sampling

Chapter 4

Data collection, analysis, conclusion and recommendation

4.1 Data collection

The data was collected from Kenya Commercial Bank, Kisumu and Post Bank, Kisumu. The data was collected using systematic sampling technique within some randomly selected hours and days of September 2019. The days were 6th , 12th, 18th, 24th and 30th of September 2019 for both the Kenya Commercial Bank and the Post Bank. The data was collected on a 1 hour basis, the time of the day being chosen randomly as shown in the appendix. The data collection on waiting times and service times was done in the same way for the two banks.

The waiting time was recorded for every fifth customer. This was the time it took the fifth customer to move in the queue until the customer had just exited the queue. The service time was measured by recording the whole period from the time the customer went to the counter to be served to the time the customer left the counter for the next customer.

4.2 Data analysis

Results for M/M/1 model at Post Bank

The average service time for five days is given by: The average service time

Table 4.1: service times

Days of september 2019	6th	12th	18th	24	30th
Average service time	3.55	3.78	3.8756	3.79	3.88

for five days is given by:

$\frac{3.55+3.78+3.875+3.79+3.88}{5}$

$= 3.775$

$= 3.78$

Average waiting time for five days =

Table 4.2: Waiting times

Days of September 2019	6th	12th	18th	24	30th
Expected waiting time	17.5	18.7	19.6	19.3	19.3

$\frac{(17.5+18.7+19.6+19.3+19.3}{3}$

$= 18.88$ minutes

The average waiting time for the Post Bank single queue single server system w1 =18.88 minutes

Results for M/M/R Model at KCB

Table 4.3: Service times for the single queue-three servers system at KCB

Days of September 2019	6th	12th	18th	24	30th
μ in minutes	5.23	4.42	4.375	3.87	4.00

Average waiting time for five days $= \frac{5.23+4.42+4.375+3.87+4.00)}{5}$

$= 4.379 minutes$

4.38 minutes

Table 4.4: Average waiting times at KCB

Days of September 2019	6th	12th	18th	24	30th
Expected waiting time/minute	6.98	7.39	7.40	6.67	6.74

Average waiting time for five days =

$\frac{(6.98+739+7.40+6.67+6.74}{5}$

= 7.04 minutes

The average waiting time for the M/M/3 queuing system at KCB ,$W_3 = 7.04 Minutes$ These results shows that $\mu_1 = 3.78 < \mu_3 = 4.38$

$W_1 = 18.88 > W_3 = 7.04$

Comparing the means of the waiting times in Post Bank M/M/1 to the Kenya commercial Bank's M/M/3 equivalence of the Post Bank.

Table 4.5: Average waiting times at KCB and Post Bank Compared

Days of September 2019	6th	12th	18th	24	30th	Averag
Average waiting at Post Bank	17.5	18.7	19.6	19.3	19.3	18.88
Average waiting at KCB	6.98	7.39	7.40	6.67	6.74	7.04
Average waiting at KCB's Post Bank equivalence	20.94	22.17	22.2	20.01	20.22	21.12

The average time in Table 4.5 in the fourth row ,(W_q), is obtained multiplying the average waiting times of row 3 by 3

The variance of waiting time in Post Bank (S_P^2):

$$\sum_{1}^{5}\left\{\frac{(X-\hat{X})^2}{n-1}\right\} \tag{4.1}$$

=0.702

Similarly, the variance of waiting time at KCB (S_K^2) is given by:

$$\sum_{1}^{5}\left\{\frac{(X-\hat{X})^2}{n-1}\right\} \tag{4.2}$$

=1.6175

Test of hypotheses

$H_0 : \mu_P = \mu_K$

$H_1 : \mu_P \neq \mu_K$

The test statistic t for unequal variance is given by:

$\frac{\hat{x}_p - \hat{x}_k}{\frac{s_p^2}{n_p} + \frac{s_p^2}{n_k}}$
$= -3.2888$

Welch's approximation formula for degrees of freedom

Welch's approximation formula for degrees of freedom is given by:

$$df \approx \frac{(\frac{s_p^2}{n_p} + \frac{s_k^2}{n_k})^2}{\sqrt{(\frac{s_p^4}{n_p^2(n_p-1)} + \frac{s_k^4}{n_k^2(n_k-1)})}} \tag{4.3}$$

=6.5

Taking the conservative value for the degrees of freedom at 6, the critical value at 5% significant level is given by:

$$t_{0.05}; 6 = 2.45 \tag{4.4}$$

Since -3.2888 is less than -2.45 or 3.2888 is greater than 2.45, we reject H_0 at 5% significant level and conclude that the difference between the means of the waiting times at Post Bank and KCB is significant.

Comparing M/M/1 to M/M/r

For M/M/1

The expected number of customers excluding those being served is given by (3.36):

$$= \frac{\lambda^2}{\mu^2(1 - \frac{\lambda}{\mu})} \tag{4.5}$$

Hence, (4.3) becomes :

$$\frac{\rho^2}{1-\rho} \tag{4.6}$$

For 2 M/M/1 model

From (3.50) the number customers in the queue excluding those being served is given by:

$$\frac{2\rho^2}{1-\rho} \tag{4.7}$$

For M/M/r

From (3.48) we obtain the number customers waiting to be served excluding those being served For r=2, the equation becomes:

$$E(N) = \frac{P_0\rho^3}{(2-1)!(2-\rho)^2} \tag{4.8}$$

$$E(N) = \frac{P_0\rho^3}{(2-\rho)^2} \tag{4.9}$$

But p_0 is given by 3.45 which solves to

$$P_0 = \frac{2-\rho}{2+\rho} \tag{4.10}$$

Hence (4.7) solves to :

$$\frac{\rho^3}{4-\rho^2} \tag{4.11}$$

When for 2M/M/1 $\lambda_1 = \lambda$, then for M/M/2 $\lambda_2 = 2\lambda$ from (4.9) the number of customers waiting to be served is given :

$$\frac{(\frac{2\lambda}{\mu})^3}{4-(\frac{2\lambda}{\mu})^2} \tag{4.12}$$

Therefore the equation becomes:

$$\frac{2\rho^3}{1-\rho^2} \tag{4.13}$$

Assuming (4.5)$\geq$ (4.11)

Equating the two and solving we obtain:

$$\rho \ = 0 \ or \ 1 \tag{4.14}$$

Since $\rho \geq 1$ makes E(N) very large, infinite or negative , ρ can take the values less one.

ρ	E(N)=	E(N')
0.1	0.02	0.022
0.2	0.017	0.100
0.3	0.59	0.257
0.4	0.152	0.533
0.5	0.333	1
0.6	0.675	1.8
0.7	1.290	3.26
0.8	2.844	6.4
0.9	7.67	16.2
1.0	∞	∞

Table 4.6: Number of customers waiting in the queue for M/M/2 and for 2M/M/1 for some values of ρ for $\mu_1 = \mu_2 = \mu$

For $\mu_1 = \frac{\mu_2}{2} = \frac{\mu}{2}$
$\rho_1 = \frac{2\lambda}{\mu}, \rho_2 = \frac{2\lambda}{2\mu}$
Which implies that
$\rho_2 = \rho, \rho_1 = 2\rho$
Hence $\frac{2(2\rho)^2}{1-2\rho}$
$= \frac{8\rho^2}{1-2\rho}$

ρ	E(N)=$\frac{2\rho^3}{1-\rho^2}$	E(N')=$\frac{8\rho^2}{1-2\rho}$
0.1	0.02	0.100
0.2	0.017	0.533
0.3	0.59	0.180
0.4	0.152	0.64
0.5	0.333	∞
0.6	0.675	0
0.7	1.290	0
0.8	2.844	0
0.9	7.67	0
1.0	∞	

Table 4.7: Number of customers waiting in the queue for M/M/2 and for 2M/M/1 for some values of ρ for $\mu_1 = \frac{\mu_2}{2} = \frac{\mu}{2}$

For $\mu_1 = 2\mu_2 = 2\mu$

$\rho_1 = \frac{\lambda}{\mu_1} = \frac{\lambda}{2\mu}, \rho_2 = \frac{\lambda}{\mu}$

This implies that, $\rho_1 = \frac{\rho}{2}$ *and* $\rho_2 = \rho$

ρ	E(N)=$\frac{2\rho^3}{1-\rho^2}$	E(N')=$\frac{\rho^2}{2-\rho}$
0.1	0.02	0.005
0.2	0.017	0.0.022
0.3	0.59	0.053
0.4	0.152	0.010
0.5	0.333	0.017
0.6	0.675	0.257
0.7	1.290	0.576
0.8	2.844	0.533
0.9	7.67	0.736
1.0	∞	1
1.5	0	4.5
1.9	0	36.1
2.0	0	∞

Table 4.8: Number of customers waiting in the queue for M/M/2 and for 2M/M/1 for some values of ρ for $\mu_1 = 2\mu_2 = 2\mu$

When $r = 3\lambda_1$ for 3 M/M/1=λ and λ_3 for M/M/3=3λ
For 3 M/M/1

$$E(N) = \frac{3\rho^2}{1-\rho} \tag{4.15}$$

For M/M/3

$$E(N) = \frac{P_0\rho^{r+1}}{(r-1)!(r-\rho)^2} \tag{4.16}$$

=

$$E(N) = \frac{P_0\rho^4}{2(3-\rho^2)} \tag{4.17}$$

This solves to:

$$E(N) = \frac{\rho^4}{(3-\rho)(\rho^2-4\rho+6} \tag{4.18}$$

=

$$E(N) = \frac{81\rho^4}{(3-3\rho)(9\rho^2+12\rho+6} \tag{4.19}$$

When $3M/M/1 \geq M/M/3$

$$\frac{3\rho^2}{1-\rho} \geq \frac{81\rho^4}{(3-3\rho)(9\rho^2+12\rho+6} \tag{4.20}$$

and

$$\frac{3\rho^2}{1-\rho} = \frac{81\rho^4}{(3-3\rho)(9\rho^2+12\rho+6} \tag{4.21}$$

This solves to

$$2\rho^2 - \rho - 1 = 0 \tag{4.22}$$

Whose real root =1

From which it can be seen that when $\rho \geq 1$ the expression for E(N)s is either infinite or negative when;

when $\mu_1 = \frac{\mu}{3} = \frac{u}{3}$ hence $\rho_1 = 3\rho$ E(N)=$\frac{3(3\rho)^2}{1-3\rho}$

$$\frac{27\rho^2}{1-3\rho} \tag{4.23}$$

ρ	E(N)=$\frac{81\rho^4}{(3-3\rho)(9\rho^2 12+6)}$	E(N')=$\frac{3\rho^2}{1-\rho}$
0.1	0.0004	0.003
0.2	0.008	0.015
0.3	0.063	0.128
0.4	0.094	0.8
0.5	0.360	1.5
0.6	0.192	2.7
0.7	1.477	4.9
0.8	2.588	9.6
0.9	7.354	24.3

Table 4.9: Number of customers waiting in the queue for M/M/3 and for 3M/M/1 for some values of ρ for $\mu_1 = \frac{\mu_2}{3} = \frac{\mu}{3}$

Table 4.10: M/M/3 and 3 /M/M/1

ρ	E(N)=$\frac{81\rho^4}{(3-3\rho)(9\rho^2 12+6)}$	E(N')=$\frac{27\rho^2}{1-3\rho}$
0.1	0.0004	0.386
0.2	0.008	2.7
0.3	0.063	0.128
0.4 hline 0.5	0.094 0.360	24.3 0
0.6	0.192	0
0.7	1.477	0
0.8	2.588	0
0.9	7.354	0

Table 4.11: Table

ρ	E(N) $= \frac{81\rho^4}{(3-3\rho)(9\rho^2+12\rho+6}$	E($\tilde{N}$) $= \frac{\rho^2}{3-\rho}$
0.1	0.0004	0.003
0.2	0.008	0.014
0.3	0.063	0.033
0.4	0.094	0.062
0.5	0.360	0.100
0.6	0.192	0.150
0.7	1.477	0.213
0.8	2.588	0.290
0.9	7.354	0.3857
1.0	∞	0.500
1.5	0.000	1.500
2.0	0.000	4.000
2.9	0.000	84.1

When $\mu_1 = 3\mu_3$ then E(N) $\geq$ E($\tilde{N}$)

4.3 Conclusion and recommendation

Conclusion

One of the objectives of our study was to determine the average waiting time a customer is likely to take in the Post Bank, Kisumu branch, which employs the M/M/1 queuing model and the average waiting time a customer is likely to take in the Kenya Commercial Bank, Kisumu branch, which employs the M/M/r queuing model.

In our study we found that the average waiting time for the M/M/1 queuing model practiced by Post Bank is 18.88 minutes while that for the Kenya Commercial Bank which practices the M/M/3 most of the time is 7.04 minutes. The equivalent of this to the Post Banks M/M/1 model is 21.12 minutes. This is larger than the waiting time in Post Bank. The difference between the means of waiting times shows that the difference is significant at the 5% Another area of our study was to compare the behavior of the M/M/1 queuing system in Post Bank with that of the M/M/r under different conditions .In doing this we identified average service times in the two banks as a suitable parameter whose changes can lead to significant changes on other performance indicators of the two queuing systems. From the study, we found that the average service times in Post Bank and Kenya Commercial Bank are 3.78 minute and 4.38 minutes respectively. This shows that the M/M/1 model as practiced by Post Bank has smaller average service times than the M/M/3 model practiced by the Kenya Commercial.

Lastly we did a theoretical comparison of the number of customers waiting in the queue to be served for the rM/M/1 and the M/M/r queuing systems. We did this by comparing the 2M/M/1 model with the M/M/2 using the same utilization factors, and also did for the case when r = 3. It can be

noted that the study gave consistent results for both cases.
From the theoretical analysis we conclude that the number of customers waiting to be served is generally larger in the rM/M1 queuing systems than in the M/M/r except when the average service time in the rM/M/ system is larger than that in the M/M/r system. Although the small average service time makes the average waiting time low, the comparison of utility factor shows that low average service time in rM/M/1 leads to low utilization of the facility hence enhance idleness. The analysis also shows that when the average service times are low and the numbers of customers waiting to be served are very large, the rM/M/1 has very high utilization factors.

Recommendations

Post Bank can still practice the M/M/1 system because of the small average service time but adopt the M/M/r model of the Kenya Commercial Bank when the number of customers waiting to be served is large.
Post Bank can adopt the rM/M1 model when the average service time is large. Post Bank should consider the low service times as an aspect that promotes idles as shown by low utilization factors when service times are small. Otherwise the small service times with a few customers waiting to be served may lead to redundancy. In this case they can embrace the M/M/r when the service times are very low and the number of customers waiting to be served is not very large.
Post Bank should maintain the rM/M/1 system when average service times for customers is large and the number of customers waiting to be served is also very large. This is favored by large utilization factors.
Kenya Commercial Bank can still maintain the M/M/r when the service

times are not very large. Otherwise the bank should adopt the rM/M/1 when the service times are small.

Kenya Commercial Bank should adopt the rM/M/1 when the service times are small and the number of customers waiting to be served is very large as this conditions are favored by large utilization factors in rM/M/1

4.4 APPENDIX1

Table 4.12: Waiting times for the fifth customer in the queue at Kenya Commercial Bank

Date	Time	Waiting time in minutes
6th	1.00-2.00	7.86
,,	,,	7.20
,,	,,	6.20
,,	,,	7.00
,,	,,	8.35
,,	,,	6.13
,,	,,	6.4
,,	,,	6.27
,,	,,	7.40
12th	11.30-12.30	6.71
,,	,,	6.71
,,	,,	6.82
,,	,,	6.83
,,	,,	6.98
,,	,,	8.57
,,	,,	7.69
,,	,,	6.69
,,	,,	8.94

Table 4.13: Waiting times for the fifth customer in the queue at Kenya Commercial Bank

Date	Time	Waiting time in minutes
18th	9.30-10.30	9.00
,,	,,	7.90
,,	,,	8.23
,,	,,	5.8
,,	,,	7.07
,,	,,	8.47
,,	,,	6.1
,,	,,	6.6
24th	10.30-11.30	5.73
,,	,,	6.07
,,	,,	5.3
,,	,,	8.1
,,	,,	7.5
,,	,,	8.87
,,	,,	6.3
,,	,,	6.63
,,	,,	5.5
30th	1.30-2.30	9.83
,,	,,	8.57
,,	,,	6.71
,,	,,	3.80
,,	,,	5.02
,,	,,	6.87
,,	,,	7.83
,,	,,	5.70

Table 4.14: Service time at KCB in September 2019

6th	12th	18th	24th	30th
4.40	8.30	5.10	3.20	7.6
3.70	1.70	3.9	2.40	2.5
1.40	5.5	6.3	5.8	2.8
4.8	2.3	2.8	4.3	2.22
3.2	4.5	4.6	3.6	10.4
7.75	6.2	4.7	5.5	2.1
4.4	4.9	5.5	4.4	1.8
1.5	7.3	6.4	7.0	1.0
4.4	3.1	3.8	2.2	1.9
1.9	4.8	7.3	3.7	3.5
3.3	3.1	4.3	2.6	3.7
3.4	4.4	3.7	1.8	2.3
5.1	5.2	6.30	3.1	2.8
1.5	0.42	5.9	0.9	2.9
4.1	8.61	3.7	2.6	3.3
5.9	2.47	5.2	3.7	4.5
6.7	3.8	4.6	6.0	3.3
2.5	4.7	2.2	3.3	1.7
2.4	5.15	1.9	2.2	2.3
3.3	3.9	0.9	0.8	5.1
3.8	3.0	3.3	3.3	1.4
6.1	6.3	5.4	5.7	2.86
2.8	4.67	3.9	4.4	2.3
5.9	1.07	3.2	2.7	2.86
6.7	5.78	6.1	4.9	2.15
2.9	3.3	4.7	5.3	3.95
2.3	3.4	5.5	8	5.6
3.4	1.11	2.7	2.3	4
4.3	6.0	3.8	4.5	3.4
1.3.3	4.19	6.2	3.2	9.8

Table 4.15: Waiting times for the fifth customer in the queue at the Post Bank

Date	Time	Waiting time in minutes
6th	10.00-11.00	9.22
,,	,,	17.8
,,	,,	21.5
12th	10.00-11.00	18.4
,,	,,	18.4
,,	,,	15.7
,,	,,	14.1
18th	,,	18.0
,,	,,	20.7
,,	,,	21.0
24th	,,	20.9
,,	10.00-11.00	15.8
,,	,,	17.9
30th	,,	21.8
,,	,,	18.2
,,	,,	18.0

Table 4.16: Service times for the fifth customer at the Post bank

6th	12th	18th	24rd	30th
2.17	4.2	2.4	3.8	5.7
1.5	4.0	1.7	3.1	4.2
5.0	3.2	4.7	4.8	3.8
4.0	4.5	4.2	5.5	5.2
3.6	2.5	5.0	3.7	2.9
2.30	1.8	4.6	2.8	1.7
2.17	5.3	3.9	4.4	4.6
3.3	2.2	5.3	2.2	4.3
4.1	4.6	4.1	3.8	2.6
3.8	3.6	2.8	2.6	5.0
4.4	6.2	4.3	4.3	3.4
4.8	3.8	1.9	5.0	2.7
6.0	4.1	6.0	4.2	4.3
2.5	3.3	4.5	3.3	3.5

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