

**APPLICATION OF MARKOV CHAIN MODEL IN CAREER
PROGRESSION OF UNIVERSITY ACADEMIC STAFF:A
CASE STUDY OF THE MOI UNIVERSITY - ELDORET,
KENYA.**

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DECLARATION

I, Ochieng Samuel Otieno duly declare that this project is my own work and has not been presented for a degree award in any other institution.

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DEDICATION

This work is dedicated my wife Yvonne, my son Jameel and my daughter Sonia, you people are my daily reminder of all that is good in this world. God bless you.

ABSTRACT

The use of Mathematical models for manpower planning has increased in recent times for better manpower planning quantitatively both in public and private sectors. In respect of organizational management, numerous previous studies have applied Markov chain models in describing title or level promotions, demotions, recruitment, withdrawals, or changes of different career development paths to confirm the actual manpower needs of an organization or predict the future manpower needs. The movements of staff within the grades or job group levels called transitions are usually the consequences of promotions or transfers between segments or wastage and recruitment into the system. In this study we determined and compared the transition rates of the academic staff of science and art faculties, the expected time taken before one attains the highest academic rank, and the absorption rates in the university. The data was collected from Moi University- Eldoret and the grades or job groups were: Tutorial Fellow, Lecturer, Senior Lecturer, Associate Professor, and full Professor. The study established that the transition rates are high at the Tutorial fellow and lecturer levels in both science and art with 67.09% and 86.31% and 86.00% and 97.53% respectively within the first ten years of employment. But it was low at 50% at senior lecturer and associate professor in the faculty of science and 63.51% and 88.69% for the same ranks in the faculty of arts. It took academic staff 19.51 years and 22.74 years in science and art respectively to attain the rank full professor.

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CHAPTER ONE

INTRODUCTION

This chapter gives the background information, objective, statement and the significance of the study.

1.1 Background of the Study

A Markov chain, named after a Russian Mathematician, Andrey Markov in 1907, is a random process that undergoes transition from one state to another on a state space. A Markov process is a random process in which the future is independent of the past, given the present. Thus, Markov processes are the natural stochastic analogs of the deterministic processes described by differential and difference equations. They form one of the most important classes of random processes. Markov models are being extensively used for analysis of manpower planning systems. Most of these models concentrate either on estimating the grade wise distribution of future manpower structure, given the existing structure and promotion policies, or on deriving policies towards promotion, given the required future structure. Manpower systems are hierarchical in nature and consists of a finite number of ordered grades for which internal movement or promotion of staff is possible from one grade to another though there is no promotion beyond the highest grade. Members of staff in the same grade have certain common characteristics and attributes (such as rank, trade, or experience) and the grades are mutually exclusive and exhaustive so that any staff must belong to one but only one grade at any time [6] Markov chain theory is one of the Mathematical tools used to investigate dynamic behaviours of a system (e.g. workforce system, financial system, health service system) is a special type of discrete-time stochastic process in which the time evolution of the system is described by a set of random variables.

Universities largely exist to perform three main core functions; advancement of knowledge through research, impacting of (acquired) knowledge through teaching (in its various

forms) and community service. These functions culminate into national development and competitive for a country and the world at large and need to be rewarded by participants. The achievement of universities objectives, to a large extent, depends on the academic staff because of the critical role they play in the teaching and learning process. Again, assert that the most attractive reward perceived by the staff in any organization is promotion and that promotion will improve the staff objective, performance and aspirations which put academics under pressure. There are many reasons for academics under pressure, but striving for academic promotion is regarded as one of the most influential factors. Academic staff transition within the ranks is understood as a movement from one academic rank to another higher rank. Academic promotions in educational institutions remain a crucial part of the development of both institutions and individuals for many reasons. It seeks to recognize and reward excellent staff. All academic staff, other than those on casual appointments, is eligible and encouraged to apply for academic promotion in universities around the world. In many instances, there are no quotas and all staff meeting the promotion standards can advance their promotion process. Contributions to teaching, research and service are all highly valued. Academic promotions criteria are basically factored on teaching, research and publications and community service, publication being a major factor. Fundamentally, global university rankings are used to compare educational performance and productivity nationally and internationally, and measure educational quality and excellence. Also, global university rankings are often used as an indication of a nation's global competitiveness, given the importance of higher education to social and economic growth and innovation. Promotion brings with it not only financial rewards but also a mark of recognition of the individual performance and acceptance by staff and students. The process of decision-making for academic staff promotion often involves criteria, such as tasks, activities, teaching, supervision, publications, research, consulting, conferencing, administration, and community service. From academic staff appraisal can also be evaluated through items, such as research articles produced, teaching method, presentation style, and involvement in university and community activities which at the end culminate to promotion of the individual. The Kenyan higher education system has been under transformation since 1963 (post-independence) with the changing system of

government. Academic profession in Kenya has also been changing under the impact of various approaches of new public management by the Ministry of Education through the Commission for University Education (CUE). The Commission for University Education in its harmonized criteria and guidelines for appointment and promotions of academic staff in Universities in Kenya 2014, developed the grading nomenclature as follows; Graduate assistant/research Assistant, Tutorial Fellow/ Junior Research Fellow, Lecturer/Research Fellow, Senior Lecturer/Research Fellow, Associate Professor, Professor, Adjunct Academic Staff, and Visiting Academic Staff. The Grading Nomenclature shall carry the rider "or equivalent". Universities may have equivalents provided for in their charters (CUE, 2014). Different categories of staff have different promotional criteria and are encouraged to seek for promotions as and when individuals are prepared. Modeling and formulating processes such as academic promotion has taken many dimensions but little is said of statistical application to this vital part of educational institutions. Considering the fact that promotion process is a form of transition that academic staffs go through, and at the end of their tenure, they shall retire on the grade of professor or non-professor, the process of academic promotions can assume a stochastic absorbing process.

1.2 Statement of the problem

There are many reasons for academics being under pressure in the Kenyan universities, but striving for academic promotion is regarded as one of the most influential factors. Academic staff progression is understood as a movement from one academic rank to a higher rank. Academic transition in educational institutions remains a crucial part of the planning and development of both institutions and individuals. It seeks to recognize and reward performing staff. All academic staff are eligible except the full professors to apply for academic promotion in the universities. In this project, we determined and compared the career progression rates in Science and Art based faculties, the rate of absorption and the duration of time taken before attaining the highest academic rank. We used data from Moi University- Kenya and the grades were: Tutorial Fellow, Lecturer, Senior Lecturer, Associate Professor, and Full Professor.

1.3 General objective of the Study

The general objective of the study was to determine the transition rates from one job group or grade to another by the university academic staff and to identify the area of specialization that is, whether it is the science or art based.

1.3.1 Specific objective of the Study

The study focused on the following specific objectives:

- i To determine and compare the career progression rates in Science and Art based faculties in the university.
- ii To determine the average time taken before attaining the highest academic rank or grade in the University
- iii To determine the absorption rates of the academic staff in the University.

1.4 Significance of the study

It is the responsibility of the Commission for University Education (CUE) to develop and review the Grading Nomenclature for the university academic staff. But it does not determine the duration of time that should be taken in a specific grade before promotion to the next rank. The purpose of this study was to determine and compare the transition rates in Science and Art faculties to identify the area of discipline with the best progression rates. The results would be useful to the university academic staff in career evaluation and the university human resources department in manpower planning and development. Particularly, the findings of the study can inform the development of policy by the Commission for University Education on review of the grading requirement for promotion and any other related need to academic progression.

1.5 Basic Concepts of the Markov Chain

Definition 1.5.1. Let $\{x_1, x_2, x_3, \dots\}$ be a sequence of random variables which assume values in discrete (finite or countable) state space S. we say that $X_t, t = 0, 1, 2, \dots$ is a

Markov chain if $P_{ij} = P_r(X_t = j | X_{t-1} = i)$ where $i, j = 0, 1, 2, \dots, n$. That is, the process starts in state one (i) and moves successfully to the next state (j) on a state space. A state space is a set of all possible states of dynamic system and each move on a state space is called a step. The sequence of random variables $\{X_1, X_2, X_3, \dots, X_n\}$ is said to have a Markov property.

1.5.1 First Order Markov Chain

A Markov Chain is said to be of the first order if the current state is solely dependent on the immediate previous state and the chances that the process is in state (i) at time (t-1) is $P_{ij} = P_r(X_t = j | X_{t-1} = i)$. Where P_{ij} can be estimated as;

$$P_{ij} = \frac{q_{ij}}{\sum_{j=1}^n q_{ij}} \quad (1.1)$$

And $i, j = 0, 1, \dots, n$, q_{ij} is the historical frequency of transition from (i) to (j) and (n) is the maximum number of states.

1.5.2 Second Order Markov Chain

The Markov property specifies that the probability of a state depends only on the probability of the previous state. But we can build more "memory" into our states by using a higher order Markov model. A Markov model is said to be second order or higher if m is greater than one and it satisfies the following conditions.

$$P_r(X_t = j | X_{t-1} = i_1, X_{t-2} = i_2, \dots, X_0 = i_0) = P_r(X_t = j | X_{t-1} = i_1, X_{t-2} = i_2) \quad (1.2)$$

Equations (1.1) and (1.2) are referred to as second order Markov chain. The equation

$$P(X_i | X_{i-1}, X_{i-2}, \dots, X_{i-m}) = P_r(X_t | X_{t-1}, \dots, X_{t-m}) \quad (1.3)$$

is referred to m^{th} order Markov chain

1.5.3 Homogenous DTMC

A Discrete-Time Markov Chain (DTMC) is said to be homogenous iff its transition probabilities do not depend on the time t, i.e.

$$P(X_{t+1} = j | X_t = i) = P(X_1 = j | X_0 = i) = P_{ij} \quad (1.4)$$

CHAPTER TWO

LITERATURE REVIEW

This Chapter focusses on the review of related literature of the study

In the respect of organizational management and manpower planning, numerous previous studies have applied the Markov chain in describing title or level promotions, demotions, or changes of different career development paths to confirm the actual manpower needs of an organization or predict the future manpower needs. Optimization of manpower and forecasting manpower needs in modern conglomerates are essential part of the future strategic planning and a very important different nature of business imperatives,[23]. The Markov chain model allows us to answer questions from policy makers. For example, it allows easy computation of various statistics at both individual and aggregate levels. At the individual level, it can be used to describe the probabilistic progression for a staff at a given career stage. At aggregate level, it can be used to derive information on overall continuation rates and separation behavior which are critical inputs in developing retention programs. Markov chain model has been widely used in different fields including Education to study students' enrolment projection both in secondary schools and tertiary institutions. Education system is comparable to a hierarchal organization in which after an academic session, three possibilities arise in the new status of the students; the students may move to the next higher class, may repeat the same class, or leave the system successfully as graduates or drop out of the system before attaining the maximum qualification [11]. Modeling the manpower management mainly concerns the prediction of future behavior of employees, [13]. Trend researchers used Markov chain model associated or integrated to describe the change of the process in light of its historical evolutions[28]. Markov chain is one of the techniques used in operations research with possibilities view that managers in organizational decision making bodies use,[29]. Manpower planning is the process by which the management determines how an organization should move from its current manpower position to its desired manpower position. Through planning, management strives to have the right number and right kinds of people, at the right

places at the right time, doing things which result in both the organization and individual receiving maximum long-run benefits. [4] defined manpower planning as the range of philosophies, tools and techniques that any organization should deploy to monitor and manage the movement of staff both in terms of numbers and profiles. These movements of staff called transitions are usually the consequences of promotions, transfer between segments or wastage and recruitment into the system. The approach to manpower policy in most Nigerian universities appears to be guided by the traditional method of putting the right number of people in the right place at the right time or arranging the suitable number of people to be allocated to various jobs usually in a hierarchal structure, [9]. Manpower planning involves two stages. The first stage is concerned with the detailed planning of manpower requirements for all types and levels of employees throughout the planning period and the second stage is concerned with the planning of manpower supplies to provide the organization with the right types of people from all sources to meet the planning requirements. An adage says, he who fails to plan, plans to fail. The planning process is one of the most crucial, complex and continuing managerial functions which embraces organizational, development, managerial development, career planning and succession planning. The process of manpower planning may rightly be regarded as a multi-step process including various issues such as; Deciding goals or objectives, Auditing of internal resources, Formulation of recruitment plan, Estimating future organizational structure and manpower requirements and developing a human resource plan. Effective manpower planning is very crucial which organizations, like large companies, academic system, federal and state administration must carry out, since human resources are considered as the most crucial, volatile and potentially unpredicted resource which an organization utilizes. The prediction of manpower is subject to how current supply of employees will change internally. These changes are observed by analyzing what happened in the past, in terms of staff retention or movement, extrapolating into the future to see what happens with the same trend of the past. Markov chain is a useful tool in prediction and has been used extensively in many areas of human endeavors. [13] presented a predictive model of numbers of employees in a hierarchal dependent- time system of human resources, incorporating subsystems that each contains grades of the same family. The

proposed model was motivated by the reality of staff development which confirms that the path evolution of each employee is usually in its family of grades.[10] used Markov chain model and job coefficient to investigate the difference of manpower status between US and Korean nuclear industry and to predict the future manpower requirements in Korea. The workforce planning, on the basis of established process, requires a good knowledge of those deployed in the establishment, as well as entry, dropout and promotion of employees in order to reach a future plan fit and desired administration in determining the future policies of the workforce system, [23] applied Markovian models and transition probability matrix to analyze the movement of the workforce in Jordan productivity companies. To achieve his aim, he collected secondary data related to workforce movement selected from annual reports of Jordanian productivity companies (potash, phosphate and pharmaceutical) for year 2004. The purpose of manpower planning is to get a better matching between manpower requirements and manpower availability[30] considered an optimization model for manpower system where vacancies are filled up by promotion and recruitment in automation system engineering private limited. They proposed a method for the determination of transition probability of promotion and recruitment vector by using Markovian theory with certain assumptions.[26] analyzed manpower data of higher learning institution using Markov chain. His objective was to design a planning model for projecting university faculty employment under alternative policies suggested by the government.[25] focused on the improved gray Markov model in human resource internal supply forecasting, so that enterprises can reasonably predict their internal human resource supply through Markov model and provide important guarantee for enterprises to develop human resources strategic planning.[5] examined the passage of staff in a faculty using one state absorbing Markov chain. He considered two cases involving regardless of staff leaving intension and staff unwillingness to leave[27].carried out studies on two graded manpower model with bulk recruitment in both grades. They assumed that the organization is having two grades and recruitment is done in both grades in bulk. [12] presented the use of Markovian model to project the future enrolment of students in a university where they removed the assumption of certain constant values in the rate of new intake by some previous authors and provided a better method for calculating the

constant value of increment in the new intake. The general objective of the study is to determine and compare the transition rates in science and art based schools and identify the area of specialization.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 Introduction

This chapter presents research design, target population, sample design and model development.

3.2 Research design

In this study we used secondary data collected from Moi University human resources department on academic staff employment for the years of 1987 - 2018 to examine the dynamics of the academic staff within the job groups or grades. We achieved this analytically through solving the transition probability matrix of the Markov Chain to evaluate academic staff progression.

3.3 Target Population

The target population for the study will be the academic staff employed by Moi University as Tutorial fellows between 1988 and 2002 in science and art faculties and their progression recorded after every five years from the date of employment to the year 2018.

3.4 The sample design

The population of the academic staff was grouped into five academic grades or ranks in respective faculties (science and art) namely; Tutorial Fellow, Lecturer, Senior Lecturer, Associate Professor, Full Professor. Which were referred to as states.

3.5 Model Development

3.5.1 Grade or Job Group Transition

The variable $p_{i,j}$ represent job group progression of the academic staff in the university. In this study, a Markov Chain model on transition was employed to compare and determine the career progression of the academic staff in science and art based faculties in Moi University. A Markov chain is a process with finite number of states or outcomes, or events in which the probability of being in a particular state at step $n+1$ depends on the state occupied at step n .

Let $S = S_1, S_2, S_3, \dots, S_r$ be the possible states and

$$\mathbf{P}_n = \begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_r \end{pmatrix} \quad (3.1)$$

The vector of probabilities of each state at step n that is, i^{th} entry of p_n is the probability that the process is in state S_i at step n . For such a probability vector,

$$p_1 + p_2 + \dots + p_i = 1. \quad (3.2)$$

Let

$P_{ij} = \Pr\{\text{state at step}(n+1) = S_j \mid \text{state at step } n = S_i\}$ A homogeneous DTMC is described by its transition matrix $P = [p_{(i,j)}] i, j \in E$. $p_{(i,j)}$ is the transition probability; probability of moving from state i to state j . In the transition probability matrix, $p_{i,j}$, where i is the initial state (job group) and j is the next state (job group). i.e.

p_{12} : Probability of a Tutorial Fellow progressing to Lecturer

p_{23} : Probability of a Lecturer progressing to Senior Lecturer

p_{34} : Probability of a Senior Lecturer progressing to an Associate Professor

p_{45} : Probability of an Associate Professor progressing to Full Professor

p_{i6} : Probability of transitional exit of service ;($i=1,2,3,4,5,6$)

The canonical form of the transition probability matrix is;

$$\mathbf{P} = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1r} \\ p_{21} & p_{22} & \cdots & p_{2r} \\ \vdots & \cdots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \vdots \\ p_{r1} & p_{r2} & \cdots & p_{rr} \end{pmatrix} \quad (3.3)$$

p_{ij} is the conditional probability of being in state S_i at step $n+1$ given that the process was in state S_j at step n . \mathbf{P} is called transition matrix. \mathbf{P} contains all conditional probabilities of the Markov Chain. \mathbf{P} can be generated as below; state n , when $n=6$

$$\mathbf{P} = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\ p_{21} & p_{22} & p_{23} & p_{24} & p_{25} & p_{26} \\ p_{31} & p_{32} & p_{33} & p_{34} & p_{35} & p_{36} \\ p_{41} & p_{42} & p_{43} & p_{44} & p_{45} & p_{46} \\ p_{51} & p_{52} & p_{53} & p_{54} & p_{55} & p_{56} \\ p_{61} & p_{62} & p_{63} & p_{64} & p_{65} & p_{66} \end{pmatrix} \quad (3.4)$$

Definition 3.5.1 (Absorbing Markov Chain). An absorbing state is one in which the probability that the process remains in that state once it enters the state is 1. A Markov chain is absorbing if it has at least one absorbing state, and if from every state it is possible to go to an absorbing state (not necessarily in one step). A Markov Chain which is not absorbing is called transient; a state i is said to be transient if, given that we start in state i there is a non-zero probability that we will never return to that state. Markov chain with t transient states and r absorbing states, the transition probability matrix \mathbf{P} , will take the following canonical form;

$$\mathbf{P} = \begin{pmatrix} Q & R \\ O & I \end{pmatrix} \quad (3.5)$$

Where; Q is a $t \times t$ matrix, q_{ij} is the probability of an element being in state i at time $t-1$ will be in state j at time t ; $i, j = 1, 2, \dots, t$; R is a non-zero $t \times r$ matrix, r_{ik} being the probability that an element in state i at time $t-1$ will be in state k at time t ; $i, j = 1, 2, \dots, t$, and $k = 1, 2, \dots, r$; O is $r \times t$ zero matrix and I is a $r \times r$ identity matrix

Definition 3.5.2 (n—Step Transition Matrix). The n- Step transition probability matrix takes the canonical form of Chapman-Kolmogorov equation;

Theorem 3.5.3 (Chapman –Kolmogorov equation). Let $P_{ij}^n = P(X_{(m+n)} = j | X_0 = i)$ be the probability of that an element is in that state j after $(m+n)$ steps from state i

$$P_{ij}^{m+n} = \sum_{k=0}^{\infty} P_{i,k}^n P_{k,j}^m, m \geq 0 \quad \text{for all } i, j \quad (3.6)$$

Then P^n will take the following canonical form;

$$\mathbf{P}^n = \begin{pmatrix} Q^n & RN \\ O & I \end{pmatrix} \quad (3.7)$$

Where $N = 1 + Q + Q^2 + Q^3 + \dots + Q^{n-1}$ It is obtained as follows;

$$\mathbf{P} = \begin{pmatrix} Q & R \\ O & I \end{pmatrix}$$

After two steps, the transition matrix \mathbf{P} is given by;

$$\mathbf{P}^2 = \begin{pmatrix} Q & R \\ O & I \end{pmatrix} \begin{pmatrix} Q & R \\ O & I \end{pmatrix} = \begin{pmatrix} Q^2 & QR + R \\ O & I \end{pmatrix} = \begin{pmatrix} Q^2 & R(I + Q) \\ O & I \end{pmatrix} \quad (3.8)$$

$$\mathbf{P}^n = \begin{pmatrix} Q^n & R(1 + Q + Q^2 + Q^3 + \dots + Q^{n-1}) \\ O & I \end{pmatrix} \quad (3.9)$$

But $N = 1 + Q + Q^2 + Q^3 + \dots + Q^{n-1}$ and therefore,

$$\mathbf{P}^n = \begin{pmatrix} Q^n & RN \\ O & I \end{pmatrix}$$

Definition 3.5.4 (The Fundamental Matrix of Markov Chain). Transition matrix of an absorbing Markov chain follows a Canonical form, which means that the transient states come first:

$$\begin{pmatrix} Q & R \\ O & I \end{pmatrix} \quad (3.10)$$

If we have t transient states and r absorbent states, then: I : is an $r \times r$ identity matrix, O : is an $r \times t$ zero matrix, R : is a nonzero $t \times r$ matrix, giving transition probabilities from transient to absorbing states, Q : is a $t \times t$ matrix, giving transition probabilities from transient to transient states

Theorem 3.5.5. For an absorbing Markov chain the matrix $I - Q$ has an inverse N . $N = 1 + Q + Q^2 + Q^3 + \dots + Q^{n-1}$. The ij -entry n_{ij} of the matrix N is the expected number of times the chain is in state s_j , given that it starts in state s_i . The initial state is counted if $i = j$.

Definition 3.5.6. For an absorbing Markov chain P , the matrix $N = (I - Q)^{-1}$ is called the fundamental matrix for P . Where, I is an identity matrix and Q is the square matrix of the transient probabilities within P . The average number of cycles that a subject resides before absorption, given a specified starting state, is estimated from fundamental matrix N . Calculating N is the matrix algebraic equivalent of taking the inverse of the transition probabilities Q .

3.5.2 Expected duration in each state

Multiplication of the number of cycles by the length of cycle gives the expected duration in each state, given the starting state. The sum of these durations gives an estimate of expected survival, conditional on starting state.

3.5.3 Absorption Probabilities

If B is a $t \times r$ matrix with entries b_{ij} and p_{ij} is the probability of absorption in state S_j given that it starts in the transient state S_i , then:

$$B = NR \tag{3.11}$$

Where, N is the fundamental matrix and R is as in the canonical form. In practicality, these do not have any units as they are the probabilities that any given transition state will go to an absorption state.

3.5.4 Variance of N

Variance of Variance of N is given by $V = N(2n - 1)$, where n is a copy of N with only the diagonal entries preserved and zeros elsewhere and is a matrix with each entry of N squared. Each element of V represents the variance of the corresponding element of N . The square root of each element of V is the standard deviation of the corresponding element of N .

CHAPTER FOUR

MODEL FITTING

4.1 Initial Transition Probabilities

Let $n_{ij}(t)$ to represent the number of academic staff in academic rank i at time $(t-5)$ who will be in academic rank j at time t and $n_i(t-5)$ to represent the number of academic staff in academic rank i at $(t-5)$, and by assuming the multinomial distribution, the transition probabilities can be estimated by;

$$p_{ij} = \frac{(n_{ij}(t))}{(n_i(t-5))}, i, j = 1, 2, 3, 4, 5, 6 \quad (4.1)$$

p_{ij} is the probability of academic staff who are in academic rank i at time $(t-5)$ who ends up in academic rank j at time t .

4.2 Notation

The notations used in the subsequent sections are as defined below;

- P_{SA} = Both science and Arts combined transition probabilities
- P_S = Faculty of Science academic staff transition probabilities
- P_A = Faculty of Arts academic staff transition probabilities
- Q_{SA} = A component of matrix P whose states are transient states representing the proportion of academic staff who proceed to their respective next states for both Arts and Science
- Q_S = A component of matrix P whose states are transient states representing the proportion of academic staff who proceed to their respective next states for faculty Science

- Q_A = A component of matrix P whose states are transient states representing the proportion of academic staff who proceed to their respective next states for faculty Arts
- N_{SA} = The fundamental matrix for combined faculties
- N_S = The fundamental matrix for the faculty of Science
- N_A = The fundamental matrix for the faculty of Arts
- B_{SA} = The matrix of absorption probabilities for the combined faculties
- B_S = The matrix of absorption probabilities for the faculty of Science
- B_A = The matrix of absorption probabilities for the faculty of Arts

4.3 Transition rates

The transition rates of the academic staff were evaluated after every five years from the date of employment as a tutorial fellow up to the year when one becomes a full professor i.e. (1988- 2018).

4.3.1 Transition rates for the combined faculties(Science and Arts)

The transition probabilities as at 2018 of the combined faculties of 32 academic staff employed as tutorial fellows between the years 1988 and 2002;

$$\mathbf{P}_{SA} = \begin{pmatrix} 0.0300 & 0.9700 & 0.0000 & 0.000 & 0.0000 & 0.000 \\ 0.0000 & 0.4200 & 0.3900 & 0.0000 & 0.0000 & 0.1900 \\ 0.0000 & 0.0000 & 0.0800 & 0.9200 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.2800 & 0.7200 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \quad (4.2)$$

From P_{SA} we obtain Q_{SA} and R_{SA} as follows;

$$\mathbf{Q}_{SA} = \begin{pmatrix} 0.0300 & 0.9700 & 0.0000 & 0.0000 \\ 0.0000 & 0.4200 & 0.3900 & 0.0000 \\ 0.0000 & 0.0000 & 0.0800 & 0.9200 \\ 0.0000 & 0.0000 & 0.0000 & 0.2800 \end{pmatrix} \quad (4.3)$$

$$\mathbf{R}_{SA} = \begin{pmatrix} 0.0000 & 0.0000 \\ 0.0000 & 0.1900 \\ 0.0000 & 0.0000 \\ 0.7200 & 0.0000 \end{pmatrix} \quad (4.4)$$

After the second period (10 years) the transition rate is given by;

$$\mathbf{P}^2 = \begin{pmatrix} Q_{SA}^2 & R_{SA}(I_{SA} + Q_{SA}) \\ O_{SA} & I_{SA} \end{pmatrix} \quad (4.5)$$

$$\begin{pmatrix} 1.0300 & 0.9700 & 0.0000 & 0.0000 \\ 0.0000 & 1.4200 & 0.3900 & 0.0000 \\ 0.0000 & 0.0000 & 1.0800 & 0.9200 \\ 0.0000 & 0.0000 & 0.0000 & 1.2800 \end{pmatrix} \begin{pmatrix} (I_{SA} + Q_{SA})R_{SA} \\ 0.0000 & 0.0000 \\ 0.0000 & 0.1900 \\ 0.0000 & 0.0000 \\ 0.7200 & 0.0000 \end{pmatrix} = \begin{pmatrix} 0.0000 & 0.18430 \\ 0.0000 & 0.26980 \\ 0.66240 & 0.0000 \\ 0.92160 & 0.00000 \end{pmatrix} \quad (4.6)$$

By using the absorbing states model, it is clear that 18.43% of those recruited as tutorial fellows exited the service while 81.57% transited to the next academic rank and none transited to professor rank after a period of ten years. For those at the lecturer level 26.98% either exited while 73.02% proceeded to the next academic rank and none to the rank of professor. For those in senior lecturer rank 33.76% exited while 66.24% moved to the next academic rank of professor. For those in associate professor level 7.84% exited while 92.16% moved to the next academic rank of professor.

4.3.2 Transition rates for the faculty of science

The transition rate for the academic staff in the faculty of science can be obtained as follows; After ten years of service as academic staff, the transition rate will be

$$\mathbf{P}^2 = \begin{pmatrix} Q_S^2 & R_S(I_S + Q_S) \\ O_S & I_S \end{pmatrix} \quad (4.7)$$

$$\begin{pmatrix} 1.1100 & 0.8900 & 0.0000 & 0.0000 \\ 0.0000 & 0.3700 & 0.2500 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \begin{pmatrix} 0.0000 & 0.0000 \\ 0.0000 & 0.3700 \\ 0.0000 & 0.0000 \\ 0.500 & 0.000 \end{pmatrix} = \begin{pmatrix} 0.0000 & 0.3293 \\ 0.0000 & 0.1369 \\ 0.5000 & 0.0000 \\ 0.5000 & 0.0000 \end{pmatrix} \quad (4.8)$$

From the matrix calculation above, 32.93% of those engaged as tutorial fellows exited the service while 67.07% transited to the next level and none transited to the rank of professor. For those in the lecturer level 13.69% exited while 86.31% proceeded to the next level. For those in senior lecturer position none exited but 50% transited to the next level while none transited to the rank of professor. And 50% of those at the associate professor level transited to professor rank.

4.3.3 Transition rates for the faculty of Arts

The transition rate for the academic staff in the faculty of arts can be obtained as follows; After ten years of service as academic staff, the transition rate will be

$$\mathbf{P}^2 = \begin{pmatrix} Q_A^2 & R_A(I_A + Q_A) \\ O_A & I_A \end{pmatrix} \quad (4.9)$$

$$(I_A + Q_A)R_A$$

$$\begin{pmatrix} 1.300 & 0.700 & 0.0000 & 0.000 \\ 0.000 & 0.130 & 0.680 & 0.000 \\ 0.000 & 0.000 & 0.270 & 0.730 \\ 0.000 & 0.000 & 0.0000 & 0.130 \end{pmatrix} \begin{pmatrix} 0.000 & 0.000 \\ 0.000 & 0.190 \\ 0.000 & 0.000 \\ 0.870 & 0.000 \end{pmatrix} = \begin{pmatrix} 0.000 & 0.1330 \\ 0.000 & 0.0247 \\ 0.6351 & 0.000 \\ 0.1131 & 0.000 \end{pmatrix} \quad (4.10)$$

From the matrix calculation above, 13.30% of those engaged as tutorial fellows exited the service while 86.70% transited to the next level and none transited to rank of professor. For those in the lecturer level 2.47% exited while 97.53% proceeded to the next level with none making it to the rank of professor. For those in senior lecturer position only 63.51% transited to the next level. Again 11.31% of those at the associate professor level transited to professor rank.

4.3.4 Comparing the Transition rates between Sciences and Arts

It is obtained by the use of the t test to compare the mean transition rate[percentages] between the faculty of science and art.

H_0 = There is no difference in the transition rates between Sciences and arts

H_1 = There is difference in the transition rates between Sciences and arts

Table 4.1: Comparison of transition rates between Sciences and Arts

Group	Transition rate in Science	Transition rate in Arts
Mean	63.345	83.9325
SD	17.2959	14.4785
SEM	8.648	7.2392
N	4	4

Intermediate values used in calculations: $t = -1.8255$

$df = 6$

Standard error of difference = 11.278

P value and statistical significance

The two-tailed P value equals 0.1177, by conventional criteria, this difference is considered to be not statistically significant since $p=0.1177 > 0.05$. The null hypothesis (H_0) is not rejected at 5% level of significance.

4.3.5 n- Step Transition Matrix

Then P^n will take the following canonical form;

$$\mathbf{P}^n = \begin{pmatrix} Q^n & RN \\ O & I \end{pmatrix}$$

Where $N = 1 + Q + Q^2 + Q^3 + \dots + Q^{n-1}$ By assuming that the maximum age that one can remain actively involved in the university as academic staff is eighty years then, that constitutes a sixteen step transition matrix as given below; 16-step transition matrix

$$\mathbf{P}^{16} = \begin{pmatrix} Q^{16} & RN \\ O & I \end{pmatrix} \quad (4.11)$$

16- Step for combined faculties

$$\mathbf{P}_{SA} = \begin{pmatrix} 0.0300 & 0.9700 & 0.0000 & 0.000 & 0.0000 & 0.000 \\ 0.0000 & 0.4200 & 0.3900 & 0.0000 & 0.0000 & 0.1900 \\ 0.0000 & 0.0000 & 0.0800 & 0.9200 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.2800 & 0.7200 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$$

Let n be $16*5 = 80$ years i.e.

$$\mathbf{P}^{((16)5)} = \begin{pmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.6724 & 0.3276 \\ 0.0000 & 0.0000 & 0.0000 & 0.000 & 0.6724 & 0.3276 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \quad (4.12)$$

16– Step for faculties of Sciences

$$\mathbf{P}_S = \begin{pmatrix} 0.1100 & 0.8900 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.038 & 0.2500 & 0.000 & 0.0000 & 0.3700 \\ 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.5000 & 0.5000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \quad (4.13)$$

Let $n=16*5$

$$\mathbf{P}^{((16)5)} = \begin{pmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.4031 & 0.5968 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.4031 & 0.5968 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \quad (4.14)$$

16– Step for faculties of Arts

$$\mathbf{P}_A = \begin{pmatrix} 0.300 & 0.7000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.1300 & 0.6800 & 0.000 & 0.0000 & 0.1900 \\ 0.0000 & 0.0000 & 0.2700 & 0.7300 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.1300 & 0.8700 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \quad (4.15)$$

Let $n=16*5$

$$\mathbf{P}^{((16)5)} = \begin{pmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.7816 & 0.2184 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.7816 & 0.2184 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix} \quad (4.16)$$

As n becomes large (above 80 years) the transient probabilities become smaller i.e as $n \rightarrow \infty$ $Q^n \rightarrow 0$. From the above matrix multiplication it is evident that, after 80 years in service all academic staff shall have been absorbed either as full professor or exited service at any given academic rank.

4.4 Expected duration in service before attaining the highest academic rank

4.4.1 The expected duration before attaining the highest academic rank for the combined faculties

The fundamental matrix N will be considered. The matrix gives the number of cycles that a subject resides in transient states before absorption, given a specified starting state. The fundamental matrix N is given as; Where $N = 1 + Q + Q^2 + Q^3 + \dots + Q^{n-1}$

For the combined faculties, it is obtained by;

$$(I_{SA} - Q_{SA}) = \begin{pmatrix} 0.9700 & -0.9700 & 0.0000 & 0.0000 \\ 0.0000 & 0.5800 & -0.0390 & 0.0000 \\ 0.0000 & 0.0000 & 0.9200 & -0.9200 \\ 0.0000 & 0.0000 & 0.0000 & 0.7200 \end{pmatrix} \quad (4.17)$$

$$(I_{SA} - Q_{SA})^{-1} = \begin{pmatrix} 1.0300 & 1.7200 & 0.7300 & 0.9300 \\ 0.0000 & 1.7200 & 0.7300 & 0.9800 \\ 0.0000 & 0.0000 & 1.0900 & 1.3900 \\ 0.0000 & 0.0000 & 0.0000 & 1.3900 \end{pmatrix} \quad (4.18)$$

By using the Beck and Pauker 1983, the expected duration is given by; $(I_{SA} - Q_{SA})^{-1}(1111)^T$ which gives:

$$\begin{pmatrix} 1.0300 & 1.7200 & 0.7300 & 0.9300 \\ 0.0000 & 1.7200 & 0.7300 & 0.9800 \\ 0.0000 & 0.0000 & 1.0900 & 1.3900 \\ 0.0000 & 0.0000 & 0.0000 & 1.3900 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = 5 \begin{pmatrix} 4.41 \\ 3.43 \\ 2.48 \\ 1.39 \end{pmatrix} = \begin{pmatrix} 22.0500 \\ 17.1500 \\ 12.4000 \\ 6.9500 \end{pmatrix} \quad (4.19)$$

This result gives the total expected duration of time an academic staff in service takes before attaining the highest academic rank. From the result, the expected duration of the academic staff in Tutorial fellow, Lecturer, senior lecturer, and Associate professor are; 22.05, 17.15, 12.40, and 6.95 years respectively.

4.4.2 The expected duration before attaining the highest academic rank in the faculty of science

The expected duration is obtained as below;

$$(I_S - Q_S) = \begin{pmatrix} 0.8900 & -0.8900 & 0.0000 & 0.0000 \\ 0.0000 & 0.6300 & -0.2500 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & -0.1000 \\ 0.0000 & 0.0000 & 0.0000 & 0.5000 \end{pmatrix} \quad (4.20)$$

$$(I_S - Q_S)^{-1} = \begin{pmatrix} 1.1240 & 1.5870 & 0.3970 & 0.794 \\ 0.0000 & 1.5870 & 0.3970 & 0.7940 \\ 0.0000 & 0.0000 & 1.0000 & 2.0000 \\ 0.0000 & 0.0000 & 0.0000 & 2.0000 \end{pmatrix} \quad (4.21)$$

By using the Beck and Pauker 1983, the expected duration is given by; $(I_S - Q_S)^{-1}(1111)^T$ which gives;

$$\begin{pmatrix} 1.1240 & 1.5870 & 0.3970 & 0.794 \\ 0.0000 & 1.5870 & 0.3970 & 0.7940 \\ 0.0000 & 0.0000 & 1.0000 & 2.0000 \\ 0.0000 & 0.0000 & 0.0000 & 2.0000 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = 5 \begin{pmatrix} 3.9020 \\ 2.7780 \\ 3.0000 \\ 2.0000 \end{pmatrix} = \begin{pmatrix} 19.5100 \\ 13.8900 \\ 15.0000 \\ 10.0000 \end{pmatrix} \quad (4.22)$$

From the result, the expected duration before one attains the highest academic rank for the Tutorial fellow, Lecturer, senior lecturer, and Associate professor are; 19.510, 13.890, 15.000, and 10.000 years respectively.

4.4.3 The expected duration before attaining the highest academic rank in the faculty of Arts

The expected duration is obtained as below;

$$(I_A - Q_A) = \begin{pmatrix} 0.700 & -0.700 & 0.0000 & 0.0000 \\ 0.0000 & 0.8700 & -0.6800 & 0.0000 \\ 0.0000 & 0.0000 & 0.73000 & -0.7300 \\ 0.0000 & 0.0000 & 0.0000 & 0.8700 \end{pmatrix} \quad (4.23)$$

$$(I_A - Q_A)^{-1} = \begin{pmatrix} 1.4290 & 1.1490 & 1.0710 & 0.8980 \\ 0.0000 & 1.1490 & 1.0710 & 0.8980 \\ 0.0000 & 0.0000 & 1.3700 & 1.1490 \\ 0.0000 & 0.0000 & 0.0000 & 1.1490 \end{pmatrix} \quad (4.24)$$

By using the Beck and Pauker 1983, the expected duration is given by; $(I_A - Q_A)^{-1}(1111)^T$ which gives;

$$\begin{pmatrix} 1.4290 & 1.1490 & 1.0710 & 0.8980 \\ 0.0000 & 1.1490 & 1.0710 & 0.8980 \\ 0.0000 & 0.0000 & 1.3700 & 1.1490 \\ 0.0000 & 0.0000 & 0.0000 & 1.1490 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = 5 \begin{pmatrix} 4.5470 \\ 3.1180 \\ 2.51900 \\ 1.1490 \end{pmatrix} = \begin{pmatrix} 22.735 \\ 15.5900 \\ 12.5980 \\ 5.7450 \end{pmatrix} \quad (4.25)$$

From the result, the expected duration before one attains the highest academic rank for the Tutorial fellow, Lecturer, senior lecturer, and Associate professor are; 22.735, 15.590, 12.598, and 5.745 years respectively.

Table 4.2: Comparison of of duration taken before obtaining the rank of Professor

Rank	Duration in years	
	Science	Arts
Tutorial fellow→Professor	19.5100	22.050
Lecturer→Professor	13.8900	15.5900
Senior Lecturer→Professor	15.0000	12.5980
Associate Professor→Professor	10.0000	5.7450

From the *table 4.2* above its evident that those at tutorial fellow,lecturer in the faculty of Science takes a shorter time to transit to full professor compared to their counterpart in the faculty of Arts whereas those in senior lecturer and associate professor takes a longer in the faculty of Science as compared to those in Arts

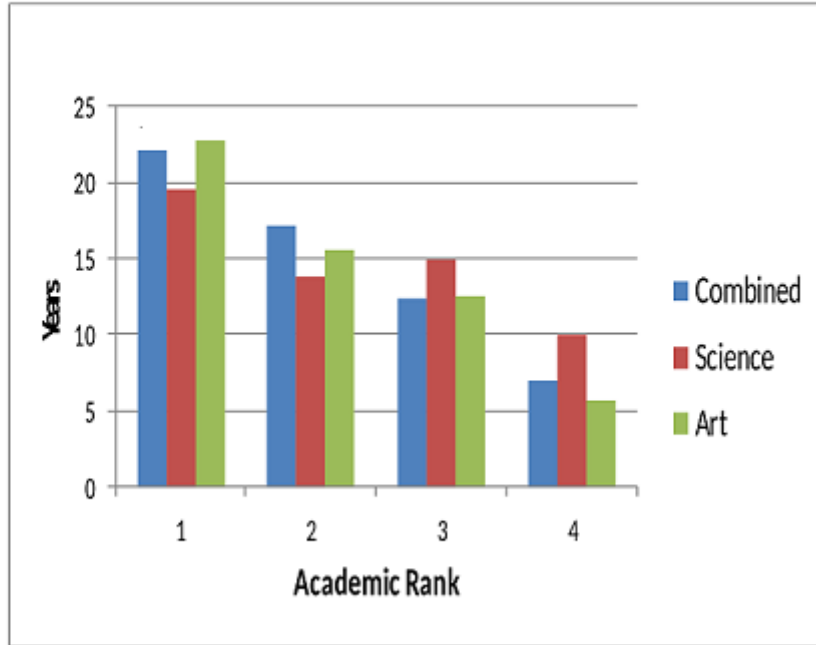


Figure 4.1: Expected duration in service before one becomes a professor

4.5 Absorption rates

4.5.1 Absorption rate for the combined faculties

$(I_{SA} - Q_{SA})^{-1}R_{SA}$ which gives;

$$B_{SA} = N_{SA}R_{SA}$$

$$\begin{pmatrix} 1.0300 & 1.7200 & 0.7300 & 0.9300 \\ 0.0000 & 1.7200 & 0.7300 & 0.9800 \\ 0.0000 & 0.0000 & 1.0900 & 1.3900 \\ 0.0000 & 0.0000 & 0.0000 & 1.3900 \end{pmatrix} \begin{pmatrix} 0.000 & .0000 \\ 0.0000 & 0.1900 \\ 0.0000 & 0.0000 \\ 0.7200 & 0.0000 \end{pmatrix} = \begin{pmatrix} 0.6724 & 0.3376 \\ 0.6724 & 0.3376 \\ 1.0000 & 0.0000 \\ 1.0000 & 0.0000 \end{pmatrix} \quad (4.26)$$

4.5.2 Absorption in the faculty of Science

$(I_S - Q_S)^{-1}R_S$ which gives;

$$B_S = N_S R_S$$

$$\begin{pmatrix} 1.1240 & 1.5870 & 0.3970 & 0.794 \\ 0.0000 & 1.5870 & 0.3970 & 0.7940 \\ 0.0000 & 0.0000 & 1.0000 & 2.0000 \\ 0.0000 & 0.0000 & 0.0000 & 2.0000 \end{pmatrix} \begin{pmatrix} 0.000 & .0000 \\ 0.0000 & 0.3700 \\ 0.0000 & 0.0000 \\ 0.5000 & 0.0000 \end{pmatrix} = \begin{pmatrix} 0.3970 & 0.6030 \\ 0.3970 & 0.6030 \\ 1.0000 & 0.0000 \\ 1.0000 & 0.0000 \end{pmatrix} \quad (4.27)$$

4.5.3 Absorption in the faculty of Arts

$(I_A - Q_A)^{-1}R_A$ which gives;

$$B_A = N_A R_A$$

$$\begin{pmatrix} 1.4290 & 1.1490 & 1.0710 & 0.8980 \\ 0.0000 & 1.1490 & 1.0710 & 0.8980 \\ 0.0000 & 0.0000 & 1.3700 & 1.1490 \\ 0.0000 & 0.0000 & 0.0000 & 1.1490 \end{pmatrix} \begin{pmatrix} 0.000 & .0000 \\ 0.0000 & 0.1700 \\ 0.0000 & 0.0000 \\ 0.8700 & 0.0000 \end{pmatrix} = \begin{pmatrix} 0.7820 & 0.2180 \\ 0.7820 & 0.2180 \\ 1.0000 & 0.0000 \\ 1.0000 & 0.0000 \end{pmatrix} \quad (4.28)$$

The result established that in the long run, absorption rates of the academic staff if the faculties are combined for the academic ranks i.e. the Tutorial fellow, Lecturer, senior lecturer, and Associate professor were; 33.76% exited the service before attaining the highest academic rank in both tutorial fellow and lecturer levels whereas 67.24% remained in service and attained the highest academic levels. For both senior lecturer and associate professor levels, all recorded 100% transition to professor level. For the faculty of science 60.30% exited or never proceeded to higher levels in tutorial fellow and lecturer whereas 39.97% attained the professor rank, senior lecturers and associate professors all achieved 100% transition to professor rank. For the faculty of arts 21.80% exited or never proceeded to the next level in both tutorial fellow and lecturer positions whereas 78.20% managed to acquire the rank of professor, senior lecturers and associate professors recorded 100% transition to the professor rank.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 Research findings

On the assumption that the academic staff can only stay for only five years in a given academic rank, a Markov chain has been used to study and analyze transition rates of academic staff in the university. From the findings, it was established that the transition rates are high at the Tutorial fellow and lecturer levels in both science and art with 67.09% and 86.31% and 86.00% and 97.53% respectively at the first five years of employment. But it was low at 50% at senior lecturer and associate professor in the faculty of science whereas in the faculty of arts the transition rates were 63.51% and 88.69% for the levels of senior lecturer and associate professor respectively. There were exits at the level of tutorial fellow and lecturer in both science and art faculties. Science recorded the highest number of exits. It was established that those academic staff in the faculty of arts stay longer before attaining the rank of professor than their colleagues in the faculty of science. The academic staff in art took 22.74 years to become full professors while those in science took 19.51 years to become full professors.

5.2 Conclusions

- i The transition rates in the academic staff in the faculty of science and the faculty arts are not significantly different and therefore it can be concluded that the transition rates are the same. The expected duration in service before an academic staff attains the rank of professor is longer in the faculty of arts than in the faculty of science.
- ii The exit rates are higher in the academic ranks of tutorial fellows and lecturer in both faculties
- iii Transition rates are higher at the ranks of senior lecturer and associate professor than at the ranks of tutorial fellow and lecturer

- iv The study has shown that transition records of promotions can be modeled as a stochastic process and the time estimates and proportion of staff at various transition levels estimated.

5.3 Recommendations

From the above conclusions, the following recommendations were made;

- i The university administration should device mechanisms on how to get hold of those recruited as tutorial fellows and lecturers to reduce the rate of exits recorded.
- ii The university administration should develop ways of encouraging the academic staff to work of their progression within the stipulated duration

5.4 Further Research

- 1 Research should be done to get into the root cause of low transition of academic staff at the tutorial fellow and lecturer levels
- 2 Research should be carried out to determine the cause of the longer time taken before attaining the rank

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