

# Modelling of Distribution of the “Matatu” Traffic Flow Using Poisson Distribution in a Highway in Kenya

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## Abstract

Traffic congestion in urban road and freeway networks leads to strong degradation of the network infrastructure and accordingly reduced output. Expansion of the available transportation continues to be one of the solution to the increasing traffic congestion, but with destruction of infrastructure. Traffic flow models are studied to be used in transport industry, in ensuring that traffic situations in our roads and highways are managed. Previous studies have modelled traffic flow by Pearson type III distribution and the Inhomogeneous Lighthill, Whitham and Richards Model (LWR) model. Research into application of Poisson to model traffic flow in the Kenyan Context is scanty. Therefore, the study models traffic flow of Thika-Nairobi highway using Poisson distribution model. The study fitted the Poisson model to a weekly traffic flow data obtained by measurement from a point method. The probability of the number of Matatu vehicles passing within the one minute period was varying and depending on the rush hours and normal hours. The parameters of the model were estimated using Analogical and Moments Method using the data from the sample. Based on Chi-square and index of dispersion values the Poisson model was identified as the adequate model for modelling traffic flow of Matatu. The observed data were used to estimate the expected data using the model. Vehicle arrivals can be evaluated by modelling arrival rate in a given interval of time and inter-arrival between the successive arrival of vehicles in a similar way.

**Keywords:** Traffic flow, Headway, Traffic volume, Matatu

# 1 Introduction

Traffic congestion and jams are not only proving to be considerably costly due to unproductive time losses, they also increase chances of accidents and environmental pollution. Some of the proposed methods of decongesting roads include: Increasing the road capacity, increasing the supply of alternative mode of transport e.g. rail transport, ferries, motorbikes and rewarding the transport of more riders per automobile. However, these methods, have challenges like waiting in bad weather and some transport mode may only benefit some topography. With the increase of the urbanization in some of major urban places like Kisumu, Thika and Nairobi there has been development of highways. Therefore, the nation needs a well developed study of traffic flow to reduce jam at the highways. Traffic flow modelling is studied in order to ease congestion on roads. Different models have been used to solve traffic flow problems, some of the commonly used models are; LWR model, Light hill ; Richards, [9], PW model, Payne; Whitham [30], Zhang second order models multi-commodity and the inhomogeneous LWR model. Homogeneous LWR model applies to traffic on a highway with no entering or exiting traffic. In reality all highways have entries and exits. The inhomogeneous LWR model takes considerations inhomogeneous factors on the road such as, junctions, number of lanes, curvature and slope among other things. It is based on conservation of traffic flow [Jin, 2000]. Pearson type III distribution for trailer ndichu [13] whose main motivation of the study was to determine the continuous impact of load on the road surface, with an aim of approximating the total weight exerted at particular time period did not consider the impact of matatus on roads but they are the major operators and even exert more weight on our roads. Furthermore, there are few trailers on highway. A few or no study has been carried using poisson modelling which requires free road of no obstacles like the highways. Therefore, the study modelled the three day operation of Matatu on Nairobi Thika highway using the data collected at a point for three consecutive days. At the highway, there is free movement of matatus with less interference like obstacles, two or more vehicles can arrive simultaneously, independently and randomly in time or space. The mean number of vehicle in an interval is directly proportional to length of interval. All these characteristics of traffic flow can be established on the highway. This places poisson model to be suitable for the traffic flow study. It is simple, robust and gives accurate results.

## 2 Review Related Literature

Research on the subject of traffic flow modeling started more than five decades ago, [36], studied high traffic flow distributions and argued that lognormal distribution should have been used for high traffic flows based on theoretical analysis with car following state but it was found that a shifted lognormal distribution gave the best fit for the data. However, the authors highlighted that Pearson type III distribution gave better results for the data but they believed that the calibration of the model is difficult and lacked explanation for the traffic process. On the other hand, normal distribution did not give any acceptable fit for any of the data sets in their study.

Ndichu and Otumba[13] carried out a study on modelling trailer-traffic flow on a Kenyan Highway by exploring the area of poor roads. They categorized traffic flow to be low, intermediate and high volume. They used the Pearson Type III distribution to cover the three states. They established that the trailers traffic flow model followed a Pearson type III distribution and recommended the model for use in predicting the number of trailers that will pass on a highway and subsequently the approximated weight exerted on the road surface by varying the time( $t$  in seconds).

Besides traffic flow distribution models that are derived from probability statistics, there are some traffic models developed in fluid mechanics. Some of the work done in this line includes the work of Burger et al[26]. He looked at the inhomogeneous traffic model and solved its p.d.e using different numerical schemes with the aim of showing their convergence. Ngonduy[28] looked at the wave propagation solution for a higher order macroscopic traffic model in the presence of inhomogeneities. He solved the p.d.e of higher order with a stiff source term. Gani[29] solved the traffic flow p.d.e. as compressible fluid. Johana et al[25] solved p.d.e governing the traffic situations using finite volume method to determine the wave propagation for traffic flow at a point when roads merges or diverges. Traffic flow models in fluid mechanics have been studied for quite some time. One of the model that has attracted less attention is the one involving inhomogeneities on the road. This has partly been attributed to the complexity involved while solving its p.d.e.

On the other hand, models associated with the properties of driver and vehicle is developed for special purpose and the data collection procedure, parameter estimation and general modeling is complex. Although some of these models consider congestion, they dont emphasis on impact of delays caused by major road operators in highway. Furthermore, few studies on the highways have been established. Congestion is also caused by irregular occurrences, such as traffic accidents, poor road network, vehicle disablement, and spilled loads and hazardous materials.

### 3 Results and Discussion

The calculated chi-square value was less than the table value i.e  $\chi_{0.05}^2$  at 11 degrees of freedom is 9.2071 so we failed to reject to the null hypothesis and conclude that the matatu traffic flow data followed a poisson distribution. Other values of chi-square at 0.01 and 0.1 are 17.275 and 22.616 respectively all lead to same conclusion.

Since the data had been established to follow a poisson distribution, the parameter of the poisson model was established as shown below:

$$\text{Average rate of arrival, } \lambda = \frac{618}{120} = 5.15$$

Further, the main results were:

(i)

$$p(X) = \frac{5.15^x e^{-5.15}}{x!} \quad x = 0, 1, 2, \dots$$

$$0 \quad \text{elsewhere}$$

(ii) The general analysis showed that the the rate of arrival was 5.15 and the confidence limits were constructed for the parameter. It was established at 99% confidence interval that limits were [4.94312 , 24.14494].

Therefore, the mean of the sample from which the population was taken was 5.15. Furthermore, we obtained rate parameter of the exponential distribution and established that the headway followed exponential distribution stated as:

$$F(t) = 0.0858e^{-0.0858t} \quad t > 0$$

### 4 Conclusion

From the general analysis the Poisson model for traffic flow was established. The study employed chi-square statistic to fit the data to the distribution. The goodness of fit test by the Chi-square statistic established that the calculated value was less than the critical value for 0.01, 0.05 and 0.1 level of significance, e.g. a  $\alpha = 0.05$ , the calculated chi-square value,  $\chi^2(9.2071)$  is less than the critical value  $\chi_{0.05}^2$  at 11 degrees of freedom ( 22.36). Therefore, we failed to reject the null hypothesis and concluded the population from which the sample data were taken was not significantly different from the distribution. The probability mass function plot was obtained and the probability mass function was positively skewed and asymptotically approaches the horizontal axis because the number of vehicle is ranging from zero to infinity. Where the rate of occurrence is very small, the range of likely possibilities will lie near the zero line. As the

rate becomes higher (as the number of matatu passing increases) the center of the curve moves towards the right and eventually, somewhere around the mean, zero occurrence actually becomes unlikely.

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