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Abstract

Seeking a clear understanding of the physical nature of quasiparticle excitations formed in the Jaynes-Cummings and antiJaynes-Cummings interaction mechanisms in the quantum Rabi model, we have introduced appropriate composite atom-field dynamical operators which characterize the algebraic structure of the quantum state space of a quasiparticle excitation. We identify the dynamical operators as generalized atom-field angular momentum operators comprising state annihilation , creation, coherence, population inversion and Casimir operators, which generate a closed generalized $SU(2)$ Lie algebra and satisfy the fermion anticommutation relations of a spinor. We therefore interpret the atom-field quasiparticle excitations in the quantum Rabi interaction as *quasiparticle spinors*. We establish that the algebraically complete quasiparticle spinor Hamiltonian automatically incorporates an intrinsic atom-field excitation number correlation operator component, yielding generalized Jaynes-Cummings Hamiltonian $\mathcal{H} = H + \hbar(\omega_0 - \omega)\hat{a}^\dagger\hat{a}s_+s_-$, antiJaynes-Cummings Hamiltonian $\overline{\mathcal{H}} = \overline{H} - \hbar(\omega_0 + \omega)\hat{a}^\dagger\hat{a}s_-s_+$ and Rabi Hamiltonian $\mathcal{H}_R = \frac{1}{2}(\mathcal{H}+\overline{\mathcal{H}})$, where H , $\overline{H}$ are the standard Jaynes-Cummings and antiJaynes-Cummings Hamiltonians. A quasiparticle spinor, identified as JC-spinor or antiJC-spinor, may be interpreted as a generalized spin-$\frac{1}{2}$ particle specified by an infinite spectrum of integer and half-integer quantum numbers and an infinite spectrum of photon-carrying spin-up and spin-down qubit states for field mode photon numbers $n = 0, 1, 2, 3, ..., \infty$. The Hamiltonian generates a general time evolving entangled state vector describing Rabi oscillations between the qubit state vectors. Expressing the time evolving state vector as a superposition of entangled state eigenvectors reveals a frequency-shift phenomenon associated with the atom-field excitation correlation energy. Considering the atom initially in a spin-up and spin-down superposition state or the field mode initially in a coherent state, the dynamical evolution of the state population inversion, coherence and the fluctuations is characterized by quantum collapses and revivals due to interference of oscillations with different Rabi frequencies.

1 Introduction

Quantized light-matter interactions are well understood to generate quasiparticle excitations generally identified as polaritons. The most basic model of fully quantized light-matter interaction is the quantum Rabi model, where a two-level atom interacts with a single mode of quantized electromagnetic field, which we revisit in this article to determine the physical nature of the quasiparticle excitations generated in the process. The standard quantum Rabi Hamiltonian is obtained in the form

$$H_R = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \hbar\omega_0 s_z + \hbar g(\hat{a} + \hat{a})(s_+ + s_-) \qquad (1a)$$

where ω , $\hat{a}$, $\hat{a}^\dagger$ are the field mode angular frequency, annihilation and creation operators, while ω_0 , s_- , s_+ , s_z are the atomic angular frequency, state lowering , raising and population inversion operators, respectively. The complete specification of the atomic state operators includes the 2×2 identity matrix I and the (Pauli) matrices $\sigma_x = s_+ + s_-$, $\sigma_y = -i(s_+ - s_-)$, which we characterize as the atomic spin coherence operators.

Noting that the field mode enters the interaction Hamiltonian in clockwise and anticlockwise rotating component forms, we apply normal and antinormal operator ordering to decompose the Rabi Hamiltonian in a two-component symmetrized form [1 , 2 , 3]

$$H_R = \frac{1}{2}(\ H\ +\ \overline{H}\ ) \qquad (1b)$$

where the normal-order component H coupling the atom to the *clockwise rotating* field mode component is the standard Jaynes-Cummings Hamiltonian taking the form

$$H = \hbar\omega\hat{a}^{\dagger}\hat{a} + \hbar\omega_0 s_z + 2\hbar g(\hat{a}^{\dagger}s_- + \hat{a}s_+) \tag{1c}$$

while the antinormal-order component $\overline{H}$ coupling the atom to the *anticlockwise rotating* field mode component is the standard antiJaynes-Cummings Hamiltonian taking the form

$$H = \hbar\omega\hat{a}\hat{a}^{\dagger} + \hbar\omega_0 s_z + 2\hbar g(\hat{a}s_- + \hat{a}^{\dagger}s_+) \tag{1d}$$

noting that the doubling of the coupling parameter $g \rightarrow 2g$ arises through the symmetrization of the Rabi Hamiltonian H_R. Hence, using $2g$ in H , $\overline{H}$ in the present context maintains consistency with the standard definition of the quantum Rabi model.

Interestingly, even though the Rabi Hamiltonian is composed of the two rotating and antirotating components, only the rotating Jaynes-Cummings component H in equation $(1c)$ has been known to generate the quasiparticle excitations characterized as polaritons, which have been widely studied in quantum optics [4 , 5 , 6 , 7]. The counterpart quasiparticle excitations generated by the antirotating antiJaynes-Cummings component $\overline{H}$, which the present author has characterized in earlier work as *antipolaritons* [1 , 2 , 3], largely remained unknown until the discovery of a conserved excitation number operator for the antiJaynes-Cummings interaction mechanism in [1 , 2 , 3], leading to exact solutions of the general dynamical evolution generated by the antiJaynes-Cummings Hamiltonian.

We observe that in [2 , 3], we reorganized the Jaynes-Cummings Hamiltonian H in equation $(1c)$ as a *polariton qubit* Hamiltonian in the form

$$H = \hbar\omega\left(\hat{N} + \frac{1}{2}\right) + 2\hbar g\hat{A} \quad ; \quad \hat{N} = \hat{a}^{\dagger}\hat{a} + s_+s_- \quad ; \quad \hat{A} = \alpha s_z + \hat{a}^{\dagger}s_- + \hat{a}s_+ \quad ; \quad \alpha = \frac{\omega_0 - \omega}{2g} \tag{1e}$$

where $\hat{N}$, $\hat{A}$ are the polariton qubit excitation number and interaction state transition operators, respectively. Similarly, we reorganized the antiJaynes-Cummings Hamiltonian $\overline{H}$ in equation $(1d)$ as an *antipolariton qubit* Hamiltonian in the form

$$\overline{H} = \hbar\omega\left(\hat{\overline{N}} - \frac{1}{2}\right) + 2\hbar g\hat{\overline{A}} \quad ; \quad \hat{\overline{N}} = \hat{a}\hat{a}^{\dagger} + s_-s_+ \quad ; \quad \hat{\overline{A}} = \overline{\alpha}s_z + \hat{a}s_- + \hat{a}^{\dagger}s_+ \quad ; \quad \overline{\alpha} = \frac{\omega_0 + \omega}{2g} \tag{1f}$$

where $\hat{\overline{N}}$, $\hat{\overline{A}}$ are the antipolariton qubit excitation number and interaction state transition operators, respectively. We note that squares of the interaction state transition operators $\hat{A}$, $\hat{\overline{A}}$ provide the corresponding excitation number operators $\hat{N}$, $\hat{\overline{N}}$ according to

$$\hat{A}^2 = \hat{N} + \frac{1}{4}\alpha^2 \quad ; \quad \hat{\overline{A}}^2 = \hat{\overline{N}}^2 + \frac{1}{4}\overline{\alpha}^2 \tag{1g}$$

The state transition operators $\hat{A}$, $\hat{\overline{A}}$ generate the respective polariton and antipolariton qubit state vectors. Simple addition and subtraction of the qubit state vectors weighted by normalization factors provide the respective polariton and antipolariton state eigenvectors and energy eigenvalues. The emerging physical picture is that, beyond their basic definition as atom-field quasiparticle excitations, polaritons and antipolaritons in the quantum Rabi model may be interpreted as photon-carrying two-state physical entities characterized by conserved excitation number and state transition operators, with quantum state spaces specified by qubit state vectors, state eigenvectors and energy eigenvalue spectra.

It is now evident that the atom-field quasiparticle excitations generated in the quantum Rabi model comprise polaritons generated in the Jaynes-Cummings interaction mechanism where the atom couples to the clockwise rotating field mode component and antipolaritons generated in the antiJaynes-Cummings interaction mechanism where the atom couples to the anticlockwise rotating field mode component. While the reformulation of the standard Jaynes-Cummings and antiJaynes-Cummings Hamiltonians H , $\overline{H}$ in equations $(1c)$, $(1d)$ into the respective polariton and antipolariton qubit Hamiltonian forms in equations $(1e)$, $(1f)$ successfully provides well defined quantum state spaces specified by qubit state vectors, state eigenvectors and energy eigenvalue spectra, the interpretation of the polaritons and antipolaritons as quantized two-state physical entities is limited by the property that the qubit Hamiltonian forms in equations $(1e)$, $(1f)$ provide only the respective excitation number and state transition operators ($\hat{N}$, $\hat{A}$) , ($\hat{\overline{N}}$, $\hat{\overline{A}}$) as the dynamical operators of a polariton or an antipolariton qubit. But, the important algebraic property that the excitation number and state transition operators as defined in equations ($1e$)-($1g$) commute with the respective Hamiltonians H , $\overline{H}$ means that their constant mean values cannot be used as order parameters for describing the dynamical evolution of the respective polariton or antipolariton qubit. In addition, just the two dynamical operators, namely the excitation number and state transition operators ($\hat{N}$, $\hat{A}$) or ($\hat{\overline{N}}$, $\hat{\overline{A}}$), alone cannot characterize the complete closed algebraic structure of the quantum state space of the polariton or antipolariton qubit. It follows immediately that the physical characterization of a polariton or an antipolariton only by its excitation number and interaction state transition operators is algebraically incomplete and we must therefore look for the appropriate dynamical operators which, besides generating the complete closed algebraic structure of the quantum state space of a polariton or an antipolariton, also provide suitable order parameters for describing the internal structure and dynamical properties of the polariton or antipolariton as a quantized physical entity.

It is important to note that in [4 , 5] and related articles, Hartmann, Brandão and Plenio have developed models of many-particle interactions in coupled arrays of cavities where the quasiparticle excitations identified as polaritons are interpreted as *bosonic particles* characterized by annihilation and creation operators defined as linear combinations of the basic atomic and field mode state lowering (annihilation) and raising (creation) operators in the form $\hat{p} = \lambda s_- + \beta \hat{a}$, $\hat{p}^\dagger = \lambda s_+ + \beta \hat{a}^\dagger$ where are real physical parameters λ , β. In this interpretation, the polariton excitation number operator is defined as $\hat{N}_p = \hat{p}^\dagger \hat{p}$ and the polariton Hamiltonian is obtained in the form $H_p = \hbar\Omega \hat{p}^\dagger \hat{p}$. We observe that substituting the annihilation and creation operators $\hat{p}$, $\hat{p}^\dagger$ reveals that the bosonic polariton Hamiltonian $H_p = \hbar\Omega \hat{p}^\dagger \hat{p}$ in [4 , 5] takes precisely the form of the Jaynes-Cummings Hamiltonian in equation ($1c$), with appropriate reorganization of physical parameters to match. The property that the annihilation-creation operator commutation bracket derived in [4 , 5] is obtained in approximate form $[\ \hat{p}\ ,\ \hat{p}^\dagger\ ] \approx 1$ means that the algebraic structure generated by the bosonic polariton excitation number, annihilation and creation operators $\hat{p}^\dagger \hat{p}$, $\hat{p}$, $\hat{p}^\dagger$ is only approximate. The bosonic polariton interpretation developed in [4 , 5] may therefore not provide the physical picture of the internal structure of the atom-field quasiparticle excitations interpreted as quantized physical entities in the general context we seek in the present work.

In this article, we advance our understanding of the internal structure and dynamical properties of quasiparticle excitations formed in atom-field interactions beyond the basic polariton and antipolariton qubit interpretation which we developed earlier in the quantum Rabi model in [2 , 3]. Considering the same Jaynes-Cummings and antiJaynes-Cummings atom-field interaction mechanisms, we now introduce a complete set of dynamical operators to determine the closed algebraic structure of the quantum state space and the physical nature of the resulting quasiparticle excitations. In contrast to the bosonic model in [4 , 5], we define the quasiparticle excitation state lowering and raising operators as composite hermitian conjugate products of the basic atom and field mode state lowering and raising operators s_- , s_+ , $\hat{a}$, $\hat{a}^\dagger$. The algebraic relations of the state lowering and raising operators provide the desired identity, state population inversion and coherence operators, where we identify the identity operator as the standard conserved excitation number operator in the Jaynes-Cummings or antiJaynes-Cummings interaction. We complete the specification of the dynamical operators by introducing the Casimir operator defined in standard algebraic form, which commutes with all the other dynamical operators and is therefore a constant of the motion of the general atom-field quasiparticle excitation Hamiltonian. The algebraic structure generated by the dynamical operators is a closed Lie algebra of the $SU(2)$ symmetry group, precisely similar to the algebraic properties of a two-state atomic spin operators, thus leading to the interpretation of the the general atom-field quasiparticle excitation as a *spinor*, in complete contrast to the bosonic polariton interpretation in [4 , 5]. The general form of the Hamiltonian which generates the quasiparticle excitation spinor is equivalent to the respective *generalized Jaynes-Cummings* or *generalized antiJaynes-Cummings* Hamiltonian, which now includes an atom-field *excitation number correlation operator* component. Because of their closed spinor algebraic properties, we call these atom-field quasiparticle excitations *Jaynes-Cummings spinors* or *antiJaynes-Cummings spinors*, depending on the atom coupling to the clockwise or anticlockwise rotating field mode component. We shall refer to a Jaynes-Cummings spinor simply as a JC-spinor and an antiJaynes-Cummings spinor as antiJC-spinor.

To develop the basic quantum state space of the composite atom-field spinor, we consider that in the Jaynes-Cummings or antiJaynes-Cummings interaction mechanism which forms a JC-spinor or an antiJC-spinor, a two-level atom initially in spin-up (excited) state $|+\rangle$ or spin-down (ground) state $|-\rangle$ is coupled to a quantized clockwise or anticlockwise rotating electromagnetic field mode in a number (Fock) state $|n\rangle$. If the atom starts in an initial spin-up state $|+\rangle$, then we identify the quasiparticle excitation formed as a *spin-up*

JC-spinor or *spin-up antiJC-spinor* with initial n-photon spin-up state vector $|\psi_{+n}\rangle$, while an interaction starting with the atom in initial spin-down state $|-\rangle$ forms a *spin-down JC-spinor* or *spin-down antiJC-spinor* with initial n-photon spin-down state vector $|\psi_{-n}\rangle$. The initial n-photon spin-up and spin-down state vectors $|\psi_{+n}\rangle$, $|\psi_{-n}\rangle$ are defined in the product form

$$|\psi_{+n}\rangle = |+\rangle|n\rangle = |+n\rangle \quad ; \quad |\psi_{-n}\rangle = |-\rangle|n\rangle = |-n\rangle \tag{1h}$$

To determine the algebraic operations of the dynamical operators on the quantum states of the JC-spinor or antiJC-spinor, we apply the standard atom and field mode operations

$$s_+|+\rangle = 0 \quad ; \quad s_+|-\rangle = |+\rangle \quad ; \quad s_-|+\rangle = |-\rangle \quad ; \quad s_-|-\rangle = 0 \quad ; \quad s_z|\pm\rangle = \pm\frac{1}{2}|\pm\rangle$$

$$\hat{a}^\dagger|n\rangle = \sqrt{n+1}\ |n+1\rangle \quad ; \quad \hat{a}|n\rangle = \sqrt{n}\ |n-1\rangle \quad ; \quad \hat{a}^\dagger\hat{a}|n\rangle = n|n\rangle \tag{1i}$$

For clarity, we develop the spin-up and spin-down JC-spinor and antiJC-spinor quantum state spaces separately in sections 2 , 3. We shall find that for each spinor, JC or antiJC, the respective spin-up and spin-down state spaces are orthogonal and independent, so that the full quantum state space of a JC-spinor or an antiJC-spinor is best composed as a simple superposition of its spin-up and spin-down state spaces where the atom starts in an initial superposition state $u_+|+\rangle + u_-|-\rangle$ so that the initial state vector $|\psi\rangle$ of the JC-spinor or antiJC-spinor is a simple superposition of the initial n-photon spin-up and spin-down state vectors $|\psi_{+n}\rangle$, $|\psi_{-n}\rangle$ obtained in the product form

$$|\psi\rangle = (u_+|+\rangle + u_-|-\rangle)|n\rangle \quad \Rightarrow \quad |\psi\rangle = u_+|\psi_{+n}\rangle + u_-|\psi_{-n}\rangle \tag{1j}$$

where $u_\pm$ are the respective atomic spin-up and spin-down state probability amplitudes. The algebraic property that the composite atom-field n-photon state vectors $|\psi_{+n}\rangle$, $|\psi_{-n}\rangle$ are orthonormal provides the initial superposition state vector $|\psi\rangle$ normalization relation

$$|u_+|^2 + |u_-|^2 = 1 \quad ; \quad \langle\psi|\psi\rangle = 1 \tag{1k}$$

We provide details of the dynamical operators and algebraic structure of the quantum state space of the JC-spinor in section 2 and the antiJC-spinor in section 3 below.

2 The Jaynes-Cummings spinor

In this section, we introduce and study the dynamical properties of a composite atom-field quasiparticle excitation generated in a Jaynes-Cummings interaction mechanism where the atom couples to the clockwise rotating field mode component. According to the form of the basic Jaynes-Cummings Hamiltonian H in equation (1c), the Jaynes-Cummings interaction mechanism which generates the quasiparticle excitations is governed by the interaction Hamiltonian component $H_I = 2\hbar g(\hat{a}^\dagger s_- + \hat{a}s_+)$. In the physical interpretation we are developing in the present work, we identify the quasiparticle excitation generated in the atom-field Jaynes-Cummings interaction mechanism as a *Jaynes-Cummings spinor*, which we shall refer to simply as a JC-spinor, thereby distinguishing it from the conventional interpretation of the atom-field quasiparticle excitations as polaritons in standard quantum optics [2-7].

It follows from the algebraic form of the Jaynes-Cummings interaction Hamiltonian $H_I = 2\hbar g(\hat{a}^\dagger s_- + \hat{a}s_+)$ that the JC-spinor may be interpreted as a composite quantized atom-field physical entity characterized by state lowering and raising operators J_- , J_+ obtained as products of the basic atom and field mode state lowering and raising operators $(s_-\ ,\ s_+)$, $(\hat{a}\ ,\ \hat{a}^\dagger)$ in the form

$$J_- = \hat{a}^\dagger s_- \quad ; \quad J_+ = \hat{a}s_+ \tag{2a}$$

Adding and subtracting J_- , J_+ provides hermitian symmetric and antisymmetric coherence dynamical operators $\hat{\Sigma}_x$, $\hat{\Sigma}_y$ in the form

$$\hat{\Sigma}_x = J_+ + J_- \quad ; \quad \hat{\Sigma}_y = -i(\ J_+ - J_-\ ) \tag{2b}$$

Using the basic atom and field mode operator algebraic relations ($I = 2\times 2$ identity matrix)

$$\hat{a}\hat{a}^\dagger - \hat{a}^\dagger\hat{a} = 1 \quad \Rightarrow \quad \hat{a}\hat{a}^\dagger = 1 + \hat{a}^\dagger\hat{a} \quad ; \quad s_-s_+ + s_+s_- = I \quad \Rightarrow \quad s_-s_+ = I - s_+s_- \tag{2c}$$

we take normal and antinormal order products of J_- , J_+ to obtain

$$J_+J_- = s_+s_- + \hat{a}^\dagger\hat{a}s_+s_- \qquad ; \qquad J_-J_+ = \hat{a}^\dagger\hat{a} - \hat{a}^\dagger\hat{a}s_+s_- \tag{2d}$$

which we add and subtract to obtain hermitian diagonal symmetric and antisymmetric dynamical operators $\hat{I}$, $\hat{\Sigma}_z$ in the form

$$\hat{I} = J_+J_- + J_-J_+ \qquad \Rightarrow \qquad \hat{I} = \hat{a}^\dagger\hat{a} + s_+s_-$$
$$\hat{\Sigma}_z = J_+J_- - J_-J_+ \qquad \Rightarrow \qquad \hat{\Sigma}_z = s_+s_- - \hat{a}^\dagger\hat{a} + 2\hat{a}^\dagger\hat{a}s_+s_- \tag{2e}$$

where we interpret $\hat{I}$ as the JC-spinor identity operator, which we identify as the excitation number operator $\hat{N}$ defined within the polariton interpretation in equation $(1e)$. The physical interpretation of the operator $\hat{\Sigma}_z$ in equation $(2e)$ follows below.

To determine the algebraic properties of these dynamical operators, we introduce basic operators J_0 , J_x , J_y , J_z consistent with the definitions of the state lowering and raising operators J_- , J_+ obtained as

$$J_0 = \frac{1}{2}\hat{I} \qquad ; \qquad J_x = \frac{1}{2}\hat{\Sigma}_x \qquad ; \qquad J_y = \frac{1}{2}\hat{\Sigma}_y \qquad ; \qquad J_z = \frac{1}{2}\hat{\Sigma}_z \tag{2f}$$

Substituting $\hat{\Sigma}_x$, $\hat{\Sigma}_y$, $\hat{I}$, $\hat{\Sigma}_z$ from equations $(2b)$, $(2e)$ then gives suitable forms

$$J_0 = \frac{1}{2}(J_+J_- + J_-J_+) \quad ; \quad J_z = \frac{1}{2}(J_+J_- - J_-J_+) \quad \Rightarrow \quad J_+J_- = J_0 + J_z \quad ; \quad J_-J_+ = J_0 - J_z$$
$$J_x = \frac{1}{2}(J_+ + J_-) \quad ; \quad J_y = -\frac{i}{2}(J_+ - J_-) \tag{2g}$$

where in defining J_0 , J_z, we take note of the explicit forms in equation $(2e)$.

Using standard algebraic property of atomic spin state lowering and raising operators s_- , s_+ gives a corresponding algebraic property of the JC-spinor state lowering and raising operators J_- , J_+ in the form

$$s_-^2 = 0 \qquad ; \qquad s_+^2 = 0 \qquad \Rightarrow \qquad J_-^2 = 0 \quad ; \quad J_+^2 = 0 \tag{3a}$$

which we apply to determine the following algebraic relations for the quadratic operators J_x^2 , J_y^2 , J_z^2 with respect to J_0 using the definitions in equation $(2g)$ in the form

$$J_x^2 = \frac{1}{2}J_0 \qquad ; \qquad J_y^2 = \frac{1}{2}J_0 \qquad ; \qquad J_z^2 = J_0^2 \tag{3b}$$

We introduce the JC-spinor Casimir operator J^2 defined in standard form by

$$J^2 = J_x^2 + J_y^2 + J_z^2 \tag{3c}$$

which on substituting J_x , J_y , J_0 from equation $(2g)$ or using the quadratic operator algebraic relations obtained in equation $(3b)$ takes the form

$$J^2 = \frac{1}{2}(J_+J_- + J_-J_+) + J_z^2 = J_0 + J_z^2 \qquad \Rightarrow \qquad J^2 = J_0(J_0+1) \tag{3d}$$

noting that substituting $2J_0 = \hat{I}$ from equation $(3b)$ provides the equivalent relation $\hat{\Sigma}_x^2 + \hat{\Sigma}_y^2 + \hat{\Sigma}_z^2 = \hat{I}(\hat{I}+2)$ usually obtained in the corresponding algebraic relations of the quantum mechanical Stokes operators in studies of polarization properties of quantized electromagnetic field modes [8 , 9 , 10].

The definition of J_z in equation $(2g)$ easily provides an important algebraic relation

$$[\ J_+ \ , \ J_- \ ] = 2J_z \tag{4a}$$

This commutation bracket relation of the state lowering and raising operators $J_\pm$, compared to the corresponding atomic spin commutation bracket $[s_+ \ , \ s_-] = 2s_z$ where s_z is the atomic spin state population inversion operator, leads us to interpret the dynamical operator J_z defined explicitly by equations $(2e)$, $(2f)$ as the JC-spinor state population inversion operator, which will be established through its algebraic operations on coupled qubit state vectors in subsection 2.2 below.

Using the definitions of J_0 , J_z , J_x , J_y , J^2 in equations $(2g)$, $(3d)$, we apply the algebraic property of J_- , J_+ in equation $(3a)$ to obtain the following closed algebraic relations, including equation $(4a)$:

$$[J_0\ ,\ J_-]=0\quad;\quad[J_0\ ,\ J_+]=0\quad;\quad[J_0\ ,\ J_x]=0\quad;\quad[J_0\ ,\ J_y]=0\quad;\quad[J_0\ ,\ J_z]=0 \tag{4b}$$

$$\{J_z\ ,\ J_-\}=0\quad;\quad\{J_z\ ,\ J_+\}=0\quad;\quad\{J_z\ ,\ J_x\}=0\quad;\quad\{J_z\ ,\ J_y\}=0\quad;\quad\{J_x\ ,\ J_y\}=0 \tag{4c}$$

$$[J^2\ ,\ J_-]=0\quad;\quad[J^2\ ,\ J_+]=0\quad;\quad[J^2\ ,\ J_0]\quad;\quad[J^2\ ,\ J_x]=0\quad;\quad[J^2\ ,\ J_y]=0\quad;\quad[J^2\ ,\ J_z]=0 \tag{4d}$$

$$[J_+\ ,\ J_-]=2J_z\quad;\quad[J_z\ ,\ J_+]=J_+\hat{I}\quad;\quad[J_z\ ,\ J_-]=-J_-\hat{I}$$
$$[J_x\ ,\ J_y]=iJ_z\quad;\quad[J_y\ ,\ J_z]=iJ_x\hat{I}\quad;\quad[J_z\ ,\ J_x]=iJ_y\hat{I} \tag{4e}$$

noting that the evaluation of the algebraic relations in equations $(4b)$-$(4d)$ using equation $(3a)$ is straightforward, while equation $(4e)$ includes a step applying equation $(2g)$ according to

$$[J_z\ ,\ J_+]=\frac{1}{2}((J_+J_-)J_++J_+(J_-J_+))=\frac{1}{2}((J_0+J_z)J_++J_+(J_0-J_z))=\frac{1}{2}(2J_+J_0+[J_z\ ,\ J_-])$$
$$[J_z\ ,\ J_-]=-\frac{1}{2}((J_-J_+)J_-+J_-(J_+J_-))=-\frac{1}{2}((J_0-J_z)J_-+J_-(J_0+J_z))=-\frac{1}{2}(2J_-J_0-[J_z\ ,\ J_-]) \tag{4f}$$

where $\hat{I}=2J_0$ as defined in equations $(2e)$, $(2f)$ is the JC-spinor identity (or conserved excitation number) operator, which commutes with $J_\pm$. We observe that equation $(4d)$ establishes the standard algebraic property that a Casimir operator commutes with all the operators which generate the closed algebra of a symmetry group. We now note that the full set of closed algebraic relations in equations $(3a)$-$(3d)$, $(4a)$-$(4e)$ constitute a generalized closed $SU(2)$ Lie algebra and the associated anticommutation relations of a spinor characterized by dynamical operators $J_\pm$, J_0 , J_z , J_x , J_y and a Casimir operator J^2. We identify this spinor as the JC-spinor, which arises as an atom-field quasiparticle excitation in a Jaynes-Cummings interaction mechanism. The JC-spinor dynamical operators are interpreted as generators of a generalized $SU(2)$ Lie algebra.

For ease of comparison with the closed $SU(2)$ Lie algebra of the basic two-state spinor (spin-$\frac{1}{2}$ particle), we present the standard closed $SU(2)$ Lie algebra generated by the atomic spin operators s_0 , s_z , s_+ , s_- easily obtained in the form

$$I=s_+s_-+s_-s_+\quad;\quad s_0=\frac{1}{2}I\quad;\quad s_+s_-=s_0+s_z\quad;\quad s_-s_+=s_0-s_z$$
$$s_-^2=0\quad;\quad s_+^2=0;\quad;\quad[s_0\ ,\ s_k]=0\quad;\quad\{s_k\ ,\ s_l\}=0\quad;\quad k\neq l=-,+,x,y,z$$
$$s_x=\frac{1}{2}(s_++s_-)\quad;\quad s_y=-\frac{i}{2}(s_+-s_-)\quad;\quad s_x^2=\frac{1}{2}s_0\quad;\quad s_y^2=\frac{1}{2}s_0\quad;\quad s_z^2=s_0^2 \tag{5a}$$

to obtain the closed algebra of the atomic spin operators in the form

$$[\ s_+\ ,\ s_-\ ]=2s_z\quad;\quad[\ s_z\ ,\ s_+\ ]=s_+I\quad;\quad[\ s_z\ ,\ s_-\ ]=-s_-I$$
$$[\ s_x\ ,\ s_y\ ]=is_z\quad;\quad[\ s_y\ ,\ s_z\ ]=is_xI\quad;\quad[\ s_z\ ,\ s_x\ ]=is_yI \tag{5b}$$

where we note that for the atomic spin case, the identity operator I is the 2×2 identity matrix giving $s_kI=s_k$, $k=\pm,x,y$, which provides the familiar form of the closed $SU(2)$ Lie algebra generated by the atomic spin operators. The atomic spin Casimir operator s^2 is obtained in the form

$$s^2=s_x^2+s_y^2+s_z^2=\frac{1}{2}(s_+s_-+s_-s_+)+s_z^2=s_0+s_z^2\qquad\Rightarrow\qquad s^2=s_0(s_0+1) \tag{5c}$$

which satisfies the standard algebraic relations for the atomic spin Casimir operator obtained in the form

$$[\ s^2\ ,\ s_0\ ]=0\quad;\quad[\ s^2\ ,\ s_\pm\ ]=0\quad;\quad[\ s^2\ ,\ s_x\ ]=0\quad;\quad[\ s^2\ ,\ s_y\ ]=0\quad;\quad[\ s^2\ ,\ s_z\ ]=0 \tag{5d}$$

The closed $SU(2)$ Lie algebra and the associated anticommutation relations of the basic two-state spinor obtained above in the set of equations $(5a)$-$(5d)$ agrees precisely in form with the closed algebraic properties generated by the JC-spinor dynamical operators in the set of equations $(3a)$-$(3d)$, $(4a$-$(4e)$, thus confirming their interpretation as generalized closed $SU(2)$ Lie algebra of a spinor.

To complete the interpretation of the JC-spinor as a quantized physical entity with internal structure and dynamical properties, we proceed to determine its Hamiltonian and quantum state space in the next two subsections.

2.1 The JC-spinor Hamiltonian

To determine the algebraic form of the JC-spinor Hamiltonian, we consider that the conserved excitation number operator is defined in symmetrized form in terms of the normal and antinormal order products of the lowering and raising operators J_- , J_+, which constitute the identity and state population inversion operators J_0 , J_z according to equations $(2d)$, $(2g)$. It follows from equation $(2d)$ that in the atom-field Jaynes-Cummings interaction mechanism which forms a JC-spinor, the normal order form $J_+J_- = s_+s_- + \hat{a}^\dagger\hat{a}s_+s_-$ provides the effective atomic spin excitation number operator, while the antinormal order form $J_-J_+ = \hat{a}^\dagger\hat{a} - \hat{a}^\dagger\hat{a}s_+s_-$ provides the effective field mode excitation number operator, each modified by the atom-field excitation number correlation operator $\pm\hat{a}^\dagger\hat{a}s_+s_-$ arising from the atom-field mode coupling as appropriate. The effective free evolution Hamiltonian of a JC-spinor is then obtained as the sum of the effective atomic spin component $\hbar\omega_0 J_+J_-$ and the field mode component $\hbar\omega J_-J_+$ in the form $\hbar\omega J_-J_+ + \hbar\omega_0 J_+J_-$, while the interaction Hamiltonian is obtained in the general form as a linear combination of the coherence components in the form $a_x\hat{\Sigma}_x + a_y\hat{\Sigma}_y$. We therefore define the JC-spinor Hamiltonian $\mathcal{H}$ in terms of the dynamical operators in the general form

$$\mathcal{H} = \hbar\omega J_-J_+ + \hbar\omega_0 J_+J_- + \hbar(a_x\hat{\Sigma}_x + a_y\hat{\Sigma}_y) \tag{6a}$$

where ω , ω_0 are the field mode and atom angular frequencies for the free evolution energy of composite atom-field system, while a_x , a_y are real physical parameters defining the atom-field interaction which forms the JC-spinor.

Substituting $\hat{\Sigma}_x = J_+ + J_-$, $\hat{\Sigma}_y = -i(J_+ - J_-)$ from equation $(2b)$ into equation $(6a)$ and symmetrizing the first two terms according to $2(aA + bB) = (a + b)(A + B) + (b - a)(B - A)$ then substituting J_0 , J_z as defined in equations $(2e)$, $(2f)$ provides the Hamiltonian in the form

$$\mathcal{H} = \hbar\overline{\delta}J_0 + \hbar\delta J_z + 2\hbar g(e^{-i\theta}J_+ + e^{i\theta}J_-) \qquad ; \qquad \overline{\delta} = \omega_0 + \omega \quad ; \quad \delta = \omega_0 - \omega \tag{6b}$$

where we have introduced blue and red sideband frequency detunings $\overline{\delta} = \omega_0 + \omega$, $\delta = \omega_0 - \omega$ and we have redefined interaction parameters to coincide with the parameter definitions in the basic Jaynes-Cummings Hamiltonian H in equation $(1c)$ for ease of comparison in the form

$$a_x \mp ia_y = |a_x \mp ia_y|e^{\mp i\theta} \qquad ; \qquad |a_x \mp ia_y| = 2g \qquad ; \qquad \tan\theta = \frac{a_y}{a_x} \tag{6c}$$

The JC-spinor Hamiltonian $\mathcal{H}$ in equation $(6b)$ takes the standard form of a spinor Hamiltonian satisfying the expected closed $SU(2)$ and anticommutation algebraic properties.

It follows from the commutation relations in equations $(4b)$, $(4d)$ that the operators J_0 , J^2 commute with the Hamiltonian $\mathcal{H}$ according to

$$[\ J_0\ ,\ \mathcal{H}\ ] = 0 \qquad ; \qquad [\ J^2\ ,\ \mathcal{H}\ ] = 0 \tag{6d}$$

which means that J_0 , J^2 are conserved dynamical operators of the JC-spinor. These operators can be used to determine the entangled state vectors which specify the quantum state space of the JC-spinor. Indeed, in the standard quantum optics literature, the operator J_0, identified as the conserved polariton excitation number operator $\hat{N} = \hat{I} = 2J_0$, has been used to determine entangled state eigenvectors and energy eigenvalues through diagonalization of the Jaynes-Cummings Hamiltonian H defined here in equation $(1c)$ with $2g \rightarrow g$. But the Casimir operator J^2 as defined in equations $(3c)$, $(3d)$, has never featured before in quantum optics, since it has only arisen here as a new conserved dynamical operator of the JC-spinor interpretation developed in the present article. The emergence of the Casimir operator J^2 and the state population inversion operator J_z determined algebraically and defined in equations $(2e)$-$(2g)$, $(4a)$ provides a standard approach for determining generalized JC-spinor state eigenvectors and eigenvalues through the generalized $SU(2)$ Lie algebra.

2.1.1 Generalized Jaynes-Cummings model : atom-field excitation correlation energy

We now determine the physical nature of the dynamics generated by the JC-spinor Hamiltonian $\mathcal{H}$. Substituting J_+J_- , J_-J_+ from equation $(2d)$ into equation $(6a)$, setting $a_y = 0$, $a_x = 2g$ and introducing the definition of $J_\mp$ from equation $(2a)$, we obtain the Hamiltonian in the form

$$a_y = 0 \quad ; \quad a_x = 2g \ : \qquad \mathcal{H} = \hbar\omega(\hat{a}^\dagger\hat{a} - \hat{a}^\dagger\hat{a}s_+s_-) + \hbar\omega_0(s_+s_- + \hat{a}^\dagger\hat{a}s_+s_-) + 2\hbar g(\hat{a}^\dagger s_- + \hat{a}s_+) \tag{7a}$$

which we reorganize in the form

$$\mathcal{H} = \hbar\omega\hat{a}^\dagger\hat{a} + \hbar\omega_0 s_+ s_- + 2\hbar g(\hat{a}^\dagger s_- + \hat{a} s_+) + \hbar(\omega_0 - \omega)\hat{a}^\dagger\hat{a} s_+ s_- \quad \Rightarrow \quad \mathcal{H} = H + \hbar(\omega_0 - \omega)\hat{a}^\dagger\hat{a} s_+ s_- \tag{7b}$$

where H is the basic Jaynes-Cummings Hamiltonian in equation $(1c)$, noting the algebraic relation $s_+s_- = s_0 + s_z \equiv s_z + \frac{1}{2}$ after writing $I \equiv 1$. It is clear in equation $(7b)$ that the JC-spinor Hamiltonian $\mathcal{H}$ is a generalization of the basic Jaynes-Cummings Hamiltonian to include an atom-field excitation number correlation operator component $\hbar(\omega_0 - \omega)\hat{a}^\dagger\hat{a}s_+s_-$, which generates an atom-field excitation correlation energy. Notice that the atom-field excitation correlation energy depends on the detuning frequency $\delta = \omega_0 - \omega$ and vanishes at resonance frequency $\omega = \omega_0$, thus meaning that experiments performed under exact resonance conditions cannot detect the atom-field excitation correlation energy. We establish below that the general dynamics generated by the JC-spinor Hamiltonian $\mathcal{H}$, which we now identify as a generalized Jaynes-Cummings Hamiltonian, is characterized by alternate field and atom blue or red frequency-shifts associated with the excitation number correlation energy.

It is important to note that the atom-field excitation number correlation operator component $\hbar(\omega_0 - \omega)\hat{a}^\dagger\hat{a}s_+s_-$ which extends the basic Jaynes-Cummings Hamiltonian H to the generalized form $\mathcal{H}$ of the JC-spinor in equation $(7b)$ is not externally introduced, but arises naturally from the algebraic property of normal and antinormal operator ordering of the products of the state lowering and raising operators J_- , J_+ taken in equation $(2d)$ to define the identity and state population inversion operators J_0 , J_z according to equations $(2e)$-$(2g)$. We may therefore treat the atom-field excitation number correlation operator, which modifies the atom and field mode excitation number operators according to equation $(2d)$, as an intrinsic dynamical property of the algebraic structure of the atom-field interaction mechanism, which does not directly manifest itself externally, thus generally missing in standard atom-field interaction models such as the quantum Rabi, Jaynes-Cummings, antiJaynes-Cummings, Dicke models, etc. We observe that in [11], such an atom-field excitation number correlation operator component has been introduced externally as a Kerr-type interaction Hamiltonian to account for phase-shift (frequency-shift) phenomena in atom-field interactions.

2.2 The JC-spinor quantum state space

Having determined all the dynamical operators, including the Casimir operator and Hamiltonian, we now complete the characterization of the internal structure of the JC-spinor by determining the state vectors and energy spectrum which specify the quantum state space of the system. As we explained earlier in section 1, we achieve clarity by developing the spin-up and spin-down JC-spinor quantum state spaces separately in subsections 2.2.1 and 2.2.2 below. We present the basic algebraic operations of the JC-spinor dynamical operators on the initial n-photon spin-up and spin-down state vectors $|\psi_{\pm n}\rangle$, which generate the respective coupled qubit state vectors, state eigenvectors and eigenvalues. The state energy eigenvalues reveal alternate atom and field mode blue or red frequency-shifts in the upper or lower spectrum.

2.2.1 Spin-up JC-spinor

As we have stated above, the initial state vector of the spin-up JC-spinor is the n-photon spin-up state vector $|\psi_{+n}\rangle$ defined in equation $(1h)$. Applying the state lowering and raising operators $J_\mp$ on $|\psi_{+n}\rangle$ in equation $(1h)$, using equation $(1i)$ and introducing the corresponding transition state vector $|\psi_{-n+1}\rangle$ to define the coupled initial spin-up JC-spinor qubit state vectors in the form

$$|\psi_{+n}\rangle = |+n\rangle \quad ; \quad |\psi_{-n+1}\rangle = |-n+1\rangle \tag{8a}$$

we obtain the coupled initial qubit state algebraic operations

$$J_-|\psi_{+n}\rangle = \sqrt{2j_{+n}}\,|\psi_{-n+1}\rangle \quad ; \quad J_+|\psi_{-n+1}\rangle = \sqrt{2j_{+n}}\,|\psi_{+n}\rangle \quad ; \quad J_+|\psi_{+n}\rangle = 0 \quad ; \quad J_-|\psi_{-n+1}\rangle = 0$$
$$j_{+n} = \frac{(n+1)}{2} \quad ; \quad n = 0, 1, 2, 3, 4, ... \tag{8b}$$

where we have introduced a quantum number j_{+n} as defined above, which takes half-integer or integer values for even or odd values of the field mode excitation (photon) number $n = 0, 1, 2, 3, 4, ...$ We call the quantum number j_{+n} the spin-up JC-spinor quantum number.

Using the algebraic operations in equation $(8b)$, we obtain eigenvalue equations generated by J_0 , J_z , J^2 as defined in equations $(2g)$, $(3d)$ in the form

$$J_0|\psi_{+n}\rangle = j_{+n}|\psi_{+n}\rangle \quad ; \quad J_0|\psi_{-n+1}\rangle = j_{+n}|\psi_{-n+1}\rangle$$

$$J_z|\psi_{+n}\rangle = j_{+n}|\psi_{+n}\rangle \quad ; \quad J_z|\psi_{-n+1}\rangle = -j_{+n}|\psi_{-n+1}\rangle$$

$$J^2|\psi_{+n}\rangle = j_{+n}(j_{+n}+1)|\psi_{+n}\rangle \quad ; \quad J^2|\psi_{-n+1}\rangle = j_{+n}(j_{+n}+1)|\psi_{-n+1}\rangle \qquad (8c)$$

The physical interpretation of J_z as the JC-spinor state population inversion operator is clearly demonstrated in the eigenvalue equations it generates on the coupled initial qubit state vectors $|\psi_{+n}\rangle$, $|\psi_{-n+1}\rangle$ in equation $(8c)$, with corresponding upper or lower eigenvalues $\pm j_{+n}$, such that we may interpret $|\psi_{+n}\rangle$ as the upper and $|\psi_{-n+1}\rangle$ as the lower initial qubit states of the spin-up JC-spinor. It is important to note that the eigenvalue equations generated by the Casimir operator J^2 take precisely the form of the standard quantum mechanical total angular momentum Casimir operator eigenvalue equations with quantum number j_{+n} taking both integer and half-integer values according to the definition in equation $(8b)$.

2.2.2 Spin-down JC-spinor

The initial state vector of the spin-down JC-spinor is $|\psi_{-n}\rangle$ defined in equation $(1h)$. Applying the state lowering and raising operators $J_{\mp}$ on $|\psi_{+n}\rangle$ in equation $(1h)$, using equation $(1i)$ and introducing transition state $|\psi_{+n-1}\rangle$ to define the coupled initial spin-down qubit states in the form

$$|\psi_{-n}\rangle = |-n\rangle \quad ; \quad |\psi_{+n-1}\rangle = |+n-1\rangle \qquad (9a)$$

we obtain coupled initial qubit state algebraic operations

$$J_-|\psi_{-n}\rangle = 0 \quad ; \quad J_+|\psi_{+n-1}\rangle = 0 \quad ; \quad J_+|\psi_{-n}\rangle = \sqrt{2j_{-n}}\ |\psi_{+n-1}\rangle \quad ; \quad J_-|\psi_{+n-1}\rangle = \sqrt{2j_{-n}}\ |\psi_{-n}\rangle$$

$$j_{-n} = \frac{n}{2} \quad ; \quad n = 0, 1, 2, 3, 4, ... \qquad (9b)$$

where, here again, we have introduced a quantum number j_{-n} as defined above, which takes zero, integer or half-integer values for zero, even or odd values of the field mode excitation (photon) number $n = 0, 1, 2, 3, 4, ...$ We interpret the quantum number j_{-n} as the spin-down JC-spinor quantum number.

Using the algebraic operations in equation $(9b)$, we obtain eigenvalue equations generated by J_0 , J_z , J^2 as defined in equations $(2g)$, $(3d)$ in the form

$$J_0|\psi_{-n}\rangle = j_{-n}|\psi_{-n}\rangle \quad ; \quad J_0|\psi_{+n-1}\rangle = j_{-n}|\psi_{+n-1}\rangle$$

$$J_z|\psi_{-n}\rangle = -j_{-n}|\psi_{-n}\rangle \quad ; \quad J_z|\psi_{+n-1}\rangle = j_{-n}|\psi_{+n-1}\rangle$$

$$J^2|\psi_{-n}\rangle = j_{-n}(j_{-n}+1)|\psi_{-n}\rangle \quad ; \quad J^2|\psi_{+n-1}\rangle = j_{-n}(j_{-n}+1)|\psi_{+n-1}\rangle \qquad (9c)$$

Here again, the physical interpretation of J_z as the JC-spinor state population inversion operator is clearly demonstrated by the eigenvalue equations it generates on the coupled initial state vectors ($|\psi_{-n}\rangle$, $|\psi_{+n-1}\rangle$) in equation $(9c)$, with corresponding lower or upper eigenvalues $\mp j_{-n}$, characterizing $|\psi_{-n}\rangle$ as the lower and $|\psi_{+n-1}\rangle$ as the upper state vector of the spin-down JC-spinor. The eigenvalue equations generated by the Casimir operator J^2 in equation $(9c)$ take the form of the standard angular momentum eigenvalue equations with quantum number j_{-n} taking zero, integer or half-integer values according to the definition in equation $(9b)$.

The basic qubit state vectors and algebraic operations in equations $(8a)$-$(8c)$, $(9a)$-$(9c)$, together with the algebraic properties in equations $(3a)$-$(3d)$, $(4a)$-$(4e)$, can be used to determine all the dynamical properties of the JC-spinor, which we now present in subsection 2.3 below.

2.3 Dynamical evolution of the JC-spinor

We now determine the internal dynamics of the JC-spinor generated by the Hamiltonian $\mathcal{H}$ expressed in final form in equation $(6b)$. As we stated earlier in section 1, it is evident in subsections 2.2.1 , 2.2.2 above that the complete quantum state space generated by the JC-spinor Hamiltonian $\mathcal{H}$ is composed of two orthogonal independent quantum spaces, depending on the initial spin-up or spin-down state of the atom, namely, the spin-up JC-spinor quantum space specified by the coupled pair of qubit state vectors ($|\psi_{+n}\rangle$, $|\psi_{-n+1}\rangle$)

defined together with algebraic operations in equations $(8a)$-$(8c)$ and the spin-down JC-spinor quantum space specified by the coupled pair of qubit state vectors $(|\psi_{-n}\rangle\ ,\ |\psi_{+n-1}\rangle)$ defined together with algebraic operations in equations $(9a)$-$(9c)$, which are orthogonal according to the relations

$$\langle\psi_{+n}|\psi_{-n}\rangle = 0 \quad ; \quad \langle\psi_{+n}|\psi_{+n-1}\rangle = 0 \quad ; \quad \langle\psi_{-n+1}|\psi_{-n}\rangle = 0 \quad ; \quad \langle\psi_{-n+1}|\psi_{+n-1}\rangle = 0 \qquad (10a)$$

Hence, instead of studying the internal dynamics of spin-up and spin-down JC-spinors separately, we find it more effective to consider the basic JC-spinor formed in initial n-photon spin-up and spin-down superposition state $|\psi\rangle$ obtained as a product of the initial atomic spin superposition state $u_+|+\rangle + u_-|-\rangle$ and the initial field mode number state $|n\rangle$. The initial JC-spinor state vector $|\psi\rangle$ is therefore a superposition of the n-photon spin-up and spin-down state vectors $|\psi_{+n}\rangle\ ,\ |\psi_{-n}\rangle$ as defined in equation $(1j)$.

Dynamical evolution of the JC-spinor described by the general time evolving state vector $|\Psi(t)\rangle$ is governed by time evolution operator $\mathcal{U}(t)$ generated by the Hamiltonian $\mathcal{H}$ through the time-dependent Schroedinger equation

$$\frac{\partial}{\partial t}|\Psi\rangle = \mathcal{H}|\Psi\rangle \qquad (10b)$$

Noting that, in general, the interaction parameters $a_x\ ,\ a_y$ which characterize the Hamiltonian in equation $(6a)\ ,\ (6b)$ may be time-dependent or time-independent, we choose the time-independent case and consider the Hamiltonian $\mathcal{H}$ to be time-independent, postponing the time-dependent form for studies of specified cases.

For the time-independent Hamiltonian $\mathcal{H}$ in equation $(6b)$, the general time evolving state vector $|\Psi(t)\rangle$ of the JC-spinor initially in the superposition state $|\psi\rangle$ is easily obtained through a simple integration of the time-dependent Schroedinger equation $(10b)$ in the form

$$|\Psi(t)\rangle = \mathcal{U}(t)|\psi\rangle \quad ; \quad \mathcal{U}(t) = e^{-\frac{i}{\hbar}\mathcal{H}t} \quad \Rightarrow \quad \mathcal{U}(t) = e^{-2igt\ (\alpha J_z + e^{-i\theta}J_+ + e^{i\theta}J_-)}\ e^{-i\bar{\delta}tJ_0} \quad ; \quad \alpha = \frac{\delta}{2g} \qquad (10c)$$

where we have applied the commutation property $[J_0\ ,\ \alpha J_z + e^{-i\theta}J_+ + e^{i\theta}J_-] = 0$ to factorize the time evolution operator $\mathcal{U}(t)$ in appropriate form for ease of evaluation. Substituting the initial state vector $|\psi\rangle$ from equation $(1j)$ into equation $(10c)$ provides $|\Psi(t)\rangle$ as a superposition of the time evolving spin-up and spin-down JC-spinor state vectors $|\Psi_{+n}(t)\rangle\ ,\ |\Psi_{-n}(t)\rangle$, respectively, in the form

$$|\Psi(t)\rangle = u_+|\Psi_{+n}(t)\rangle + u_-|\Psi_{-n}(t)\rangle \quad ; \quad |\Psi_{+n}(t)\rangle = \mathcal{U}(t)|\psi_{+n}\rangle \quad ; \quad |\Psi_{-n}(t)\rangle = \mathcal{U}(t)|\psi_{-n}\rangle \qquad (10d)$$

Substituting $\mathcal{U}(t)$ from equation $(10c)$ into equation $(10d)$ and applying the J_0 eigenvalue equations from $(8c)\ ,\ (9c)$, we express the time evolving spin-up and spin-down JC-spinor state vectors $|\Psi_{+n}(t)\rangle\ ,\ |\Psi_{-n}(t)\rangle$ in the form

$$|\Psi_{+n}(t)\rangle = e^{-i\bar{\delta}j_{+n}t}e^{-2igt\ (\alpha J_z + e^{-i\theta}J_+ + e^{i\theta}J_-)}\ |\psi_{+n}\rangle$$
$$|\Psi_{-n}(t)\rangle = e^{-i\bar{\delta}j_{-n}t}e^{-2igt\ (\alpha J_z + e^{-i\theta}J_+ + e^{i\theta}J_-)}\ |\psi_{-n}\rangle \qquad (10e)$$

Expanding the interaction time evolution operator in even and odd power terms, writing $(-i)^{2k} = (-1)^k$, $(-i)^{2k+1} = -i(-1)^k$ and using the algebraic relations from equations $(3a)\ ,\ (3b)\ ,\ (4c)$ giving

$$(\alpha J_z + e^{-i\theta}J_+ + e^{i\theta}J_-)^{2k} = (\alpha^2 J_0^2 + 2J_0)^k$$
$$(\alpha J_z + e^{-i\theta}J_+ + e^{i\theta}J_-)^{2k+1} = (\alpha^2 J_0^2 + 2J_0)^k\ (\alpha J_z + e^{-i\theta}J_+ + e^{i\theta}J_-) \quad ; \quad k = 0,1,2,... \qquad (10f)$$

we apply the respective qubit state algebraic operations generated by $J_\pm\ ,\ J_0$ in equations $(8b)\ ,\ (8c)\ ,\ (9b)\ ,\ (9c)$ and introduce standard trigonometric function expansions to obtain the time evolving spin-up and spin-down JC-spinor state vectors in equation $(10e)$ in the explicit form

$$|\Psi_{+n}(t)\rangle = e^{-i\bar{\delta}j_{+n}t}(\ (\cos(\mathcal{R}_{+n}t) - i\ c_{+n}\sin(\mathcal{R}_{+n}t)\ )|\psi_{+n}\rangle - i\ s_{+n}e^{i\theta}\sin(\mathcal{R}_{+n}t)|\psi_{-n+1}\rangle)$$
$$|\Psi_{-n}(t)\rangle = e^{-i\bar{\delta}j_{-n}t}(\ (\cos(\mathcal{R}_{-n}t) + i\ c_{-n}\sin(\mathcal{R}_{-n}t)\ )|\psi_{-n}\rangle - i\ s_{-n}e^{-i\theta}\sin(\mathcal{R}_{-n}t)|\psi_{+n-1}\rangle) \qquad (10g)$$

where we have introduced the respective Rabi oscillation frequencies $\mathcal{R}_{\pm n}$ and interaction parameters $c_{\pm n}$, $s_{\pm n}$ obtained as

$$\mathcal{R}_{\pm n} = 2g\sqrt{\alpha^2 j_{\pm n}^2 + 2j_{\pm n}} \quad ; \quad c_{\pm n} = \frac{2g\alpha j_{\pm n}}{\mathcal{R}_{\pm n}} \quad ; \quad s_{\pm n} = \frac{2g\sqrt{2j_{\pm n}}}{\mathcal{R}_{\pm n}} \quad ; \quad \alpha = \frac{\delta}{2g} \qquad (10h)$$

where the frequency detuning $\delta = \omega_0 - \omega$ has been defined in equation ($6b$). We easily determine orthonormalization relations of the general time evolving JC-spinor state vectors in the form

$$\langle\Psi_{+n}(t)|\Psi_{+n}(t)\rangle = 1 \quad ; \quad \langle\Psi_{+n}(t)|\Psi_{-n}(t)\rangle = 0 \quad ; \quad \langle\Psi_{-n}(t)|\Psi_{+n}\rangle = 0 \quad ; \quad \langle\Psi_{-n}(t)|\Psi_{-n}(t)\rangle = 1$$

$$|u_+|^2 + |u_-|^2 = 1 \qquad ; \qquad \langle\Psi(t)|\Psi(t)\rangle = 1 \tag{10i}$$

Substituting $|\Psi_{\pm n}(t)\rangle$ from equation ($10g$) into equation ($10d$) provides the general time evolving state vector $|\Psi(t)\rangle$ of the JC-spinor. We can now use the time evolving state vector $|\Psi(t)\rangle$ to determine the dynamical and statistical properties of the JC-spinor, noting that setting $u_- = 0$ or $u_+ = 0$ in equation ($10d$) gives separate descriptions of the spin-up or spin-down JC-spinor provided by the respective time evolving state vectors $|\Psi_{+n}(t)\rangle$, $|\Psi_{-n}(t)\rangle$.

2.3.1 JC-spinor state eigenvectors, energy eigenvalues and frequency-shifts

We determine an important dynamical feature of the JC-spinor by using the trigonometric relations $2\cos(\mathcal{R}_{\pm n}t) = e^{i\mathcal{R}_{\pm n}t} + e^{-i\mathcal{R}_{\pm n}t}$, $2i\sin(\mathcal{R}_{\pm n}t) = e^{i\mathcal{R}_{\pm n}t} - e^{-i\mathcal{R}_{\pm n}t}$, to reorganize the time evolving spin-up and spin-down state vectors $|\Psi_{\pm n}(t)\rangle$ in equation ($10g$) in the equivalent form

$$|\Psi_{\pm n}(t)\rangle = e^{-\frac{i}{\hbar}\mathcal{E}^{+}_{\pm n}t}|\Psi^{+}_{\pm n}\rangle + e^{-\frac{i}{\hbar}\mathcal{E}^{-}_{\pm n}t}|\Psi^{-}_{\pm n}\rangle \tag{11a}$$

where we have introduced the respective spin-up and spin-down JC-spinor state eigenvectors and energy eigenvalues $(|\Psi^{+}_{\pm n}\rangle\ ,\ \mathcal{E}^{+}_{\pm n})$, $(|\Psi^{-}_{\pm n}\rangle\ ,\ \mathcal{E}^{-}_{\pm n})$ obtained in the form

$$|\Psi^{+}_{\pm n}\rangle = \frac{1}{2}(\ (1 \pm c_{\pm n})|\psi_{\pm n}\rangle + s_{\pm n}e^{\pm i\theta}|\psi_{\mp n\pm 1}\rangle\ ) \qquad ; \qquad \mathcal{E}^{+}_{\pm n} = \hbar(\ \overline{\delta} j_{\pm n} + \mathcal{R}_{\pm n})$$

$$|\Psi^{-}_{\pm n}\rangle = \frac{1}{2}(\ (1 \mp c_{\pm n})|\psi_{\pm n}\rangle - s_{\pm n}e^{\pm i\theta}|\psi_{\mp n\pm 1}\rangle\ ) \qquad ; \qquad \mathcal{E}^{-}_{\pm n} = \hbar(\ \overline{\delta} j_{\pm n} - \mathcal{R}_{\pm n}) \tag{11b}$$

These are eigenvectors of the JC-spinor Hamiltonian $\mathcal{H}$ satisfying eigenvalue equations

$$\mathcal{H}|\Psi^{+}_{\pm n}\rangle = \mathcal{E}^{+}_{\pm n}|\Psi^{+}_{\pm n}\rangle \qquad ; \qquad \mathcal{H}|\Psi^{-}_{\pm n}\rangle = \mathcal{E}^{-}_{\pm n}|\Psi^{-}_{\pm n}\rangle \tag{11c}$$

In equation ($11b$), we express the Rabi frequency $\mathcal{R}_{\pm n}$ defined in equation ($10h$) in the form

$$\mathcal{R}_{\pm n} = 2g\alpha j_{\pm n}\lambda_{\pm n} \qquad ; \qquad \lambda_{\pm n} = \sqrt{1 + \frac{2}{\alpha^2 j_{\pm n}}} \tag{11d}$$

and substitute $\overline{\delta} = \omega_0 + \omega$, $2g\alpha = \omega_0 - \omega$ to reorganize the energy eigenvalues in the final form

$$\mathcal{E}^{+}_{\pm n} = \hbar j_{\pm n}(\ \omega(1 - \lambda_{\pm n}) + \omega_0(1 + \lambda_{\pm n})\ ) \qquad ; \qquad \mathcal{E}^{-}_{\pm n} = \hbar j_{\pm n}(\ \omega(1 + \lambda_{\pm n}) + \omega_0(1 - \lambda_{\pm n})\ ) \tag{11e}$$

which reveals that the dynamical evolution of the JC-spinor is characterized by alternate atom and field mode frequency-shifts towards the blue or red sidebands in the upper or lower energy spectrum. The frequency-shift is associated with the atom-field excitation correlation energy generated by the excitation number correlation operator component $\hbar(\omega_0 - \omega)\hat{a}^\dagger\hat{a}s_+s_-$ of the JC-spinor Hamiltonian. Since it depends on the atom-field mode frequency detuning $\delta = \omega_0 - \omega$, the excitation number correlation operator vanishes at resonance $\omega = \omega_0$, such that the frequency-shift phenomenon is not observable under resonance conditions.

2.3.2 JC-spinor state population inversion and fluctuations

We determine the JC-spinor state population inversion $Z(t)$ and mean square population inversion $\overline{Z^2}(t)$ as the mean values of the state population inversion operator J_z and its square J_z^2 in the general time evolving state $|\Psi(t)\rangle$ obtained as

$$Z(t) = \langle\Psi(t)|J_z|\Psi(t)\rangle \qquad ; \qquad \overline{Z^2}(t) = \langle\Psi(t)|J_z^2|\Psi(t)\rangle \tag{12a}$$

Using $|\Psi_{\pm n}(t)\rangle$ from equation ($10g$), we substitute $|\Psi(t)\rangle$ from equation ($10d$) into equation ($11a$), expand as appropriate, then use the J_z algebraic relations from equations ($8c$) , ($9c$) and apply the standard $|\psi_{\pm n}\rangle$,

$|\psi_{\mp n\pm 1}\rangle$ orthonormalization relations to obtain the population inversion and mean square population inversion in the final form

$$Z(t) = |u_+|^2 Z_{+n}(t) + |u_-|^2 Z_{-n}(t) \qquad ; \qquad \overline{Z^2}(t) = |u_+|^2 \ \overline{Z^2_{+n}}(t) + |u_-|^2 \ \overline{Z^2_{-n}}(t) \tag{12b}$$

where we have identified the spin-up and spin-down JC-spinor state population inversion $Z_{\pm n}(t)$ and mean square population inversion $\overline{Z^2_{\pm n}}(t)$ determined in the respective time evolving states $|\Psi_{\pm n}(t)\rangle$ in the form

$$Z_{\pm n}(t) = \langle \Psi_{\pm n}(t)|J_z|\Psi_{\pm n}(t)\rangle \qquad ; \qquad \overline{Z^2_{\pm n}}(t) = \langle \Psi_{\pm n}(t)|J_z^2|\Psi_{\pm n}(t)\rangle \tag{12c}$$

Substituting $|\Psi_{\pm n}(t)\rangle$ from equation $(10g)$ into equation $(12c)$, using equations $(8c)$, $(9c)$ and applying the appropriate orthonormalization relations as explained above, we obtain the spin-up and spin-down JC-spinor state population inversion and mean square population inversion in the explicit form

$$Z_{\pm n}(t) = \pm j_{\pm n}(\ \cos^2(\ \mathcal{R}_{\pm n}t) + (c^2_{\pm n} - s^2_{\pm n}) \sin^2(\mathcal{R}_{\pm n}t) \ ) \qquad ; \qquad \overline{Z^2}_{\pm n}(t) = j^2_{\pm n} \tag{12d}$$

We obtain the JC-spinor state population inversion fluctuations from the variance according to the definition

$$(\Delta Z(t))^2 = \overline{Z^2}(t) - (Z(t))^2 \tag{12e}$$

which on substituting the explicit forms of $Z(t)$, $\overline{Z^2}(t)$ obtained in equation $(12b)$ and reorganizing as appropriate using the relation $|u_+|^2 + |u_-|^2 = 1$ takes the form

$$(\Delta Z(t))^2 = |u_+|^2(\Delta Z_{+n}(t))^2 + |u_-|^2(\Delta Z_{-n}(t))^2 + |u_+|^2|u_-|^2(Z_{+n}(t) - Z_{-n}(t) \ )^2 \tag{12f}$$

where the spin-up and spin-down JC- spinor state population inversion fluctuations $(\Delta Z_{\pm n}(t))^2$ are obtained from the respective variances

$$(\Delta Z_{\pm n}(t))^2 = \overline{Z^2_{\pm n}}(t) - (Z_{\pm n}(t))^2 \tag{12g}$$

which on substituting the results from equation $(12d)$ take the explicit form

$$(\Delta Z_{\pm n}(t))^2 = j^2_{\pm n} \left(\ 1 - (\cos^2(\mathcal{R}_{\pm n}t) + (c^2_{\pm n} - s^2_{\pm n}) \sin^2(\mathcal{R}_{\pm n}t))^2 \ \right) \tag{12h}$$

Substituting the explicit results from equations $(12d)$, $(12h)$ into equations $(12b)$, $(12f)$ as appropriate, we obtain the JC-spinor population inversion $Z(t)$ and population inversion fluctuations $(\Delta Z(t))^2$ in the general time evolving state $|\Psi(t)\rangle$ in explicit form.

Setting the probability amplitudes $u_+ = u_- = \frac{1}{\sqrt{2}}$, we have plotted the population inversion $Z(t)$ against scaled time $\tau = gt$ in Fig.1 , Fig.2 for photon numbers $n = 1, 16$ as specified together with the parameter values $\alpha = \frac{1}{1.31}$, $\theta = 0$ which we have chosen arbitrarily, noting that $\theta = 0$ coincides with the coupling parameter definitions in the quantum Rabi Hamiltonian in equations $(1a)$-$(1d)$:

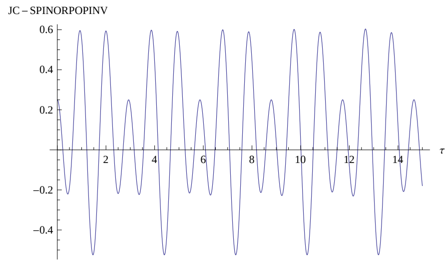

Figure 1: JC-spinor population inversion $Z(\tau)$, $\tau = gt$: $\quad \alpha = \frac{1}{1.31}$; $\theta = 0$; $n = 1$

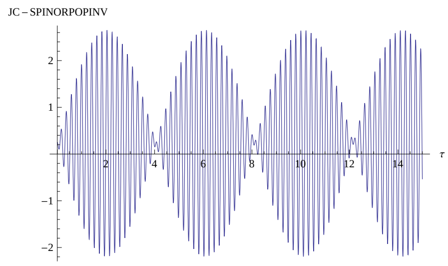

Figure 2: JC-spinor population inversion $Z(\tau)$, $\tau = gt$: $\quad \alpha = \frac{1}{1.31}$; $\theta = 0$; $n = 16$

In general, the dynamical evolution of the JC-spinor state population inversion in Fig.1 , Fig.2 takes a regular pattern, rising and falling between peaks, which may be interpreted as quantum collapses and revivals due to interference effects associated with the competing Rabi oscillation frequencies $\mathcal{R}_{+n}$, $\mathcal{R}_{-n}$ in the spin-up and spin-down components of the dynamical evolution of the spinor. We observe that the pattern of evolution in Fig.1 , Fig.2 is similar to the evolution of the atomic spin state population inversion or the field mode photon number obtained in [12 , 13], where the field mode is initially in a thermal or coherent state, with the atom starting in either spin-up (excited) or spin-down (ground) state.

As in [12 , 13], we demonstrate the phenomenon of quantum collapses and revivals in the JC-spinor in a more pronounced manner by considering a spin-up JC-spinor with the atom initially in spin-up state $|+\rangle$ and the field mode initially in a coherent state $|\beta\rangle = e^{-\frac{1}{2}|\beta|^2} \sum_{n=0}^{\infty} \frac{\beta^n}{\sqrt{n!}} |n\rangle$, such that the initial state of the spin-up JC-spinor is $|\psi_{+\beta n}\rangle = e^{-\frac{1}{2}|\beta|^2} \sum_{n=0}^{\infty} \frac{\beta^n}{\sqrt{n!}} |\psi_{+n}\rangle$. The time evolving state vector of the spin-up JC-spinor generated by the Hamiltonian $\mathcal{H}$ is then obtained in the form $|\Psi_{+\beta n}(t)\rangle = e^{-\frac{1}{2}|\beta|^2} \sum_{n=0}^{\infty} \frac{\beta^n}{\sqrt{n!}} |\Psi_{+n}(t)\rangle$, where $|\Psi_{+n}(t)\rangle$ has been determined explicitly in equation $(10g)$. We obtain the spin-up JC-spinor state population inversion in the final form

$$Z_{+\beta n}(t) = e^{-|\beta|^2} \sum_{n=0}^{\infty} \frac{|\beta|^{2n}}{n!} Z_{+n}(t) \qquad (12i)$$

where $Z_{+n}(t)$ has been evaluated explicitly in equation $(12d)$. We have plotted the population inversion $Z_{+\beta n}(\tau)$ against scaled time $\tau = gt$ in Fig.3 for the detuned case with parameter $\alpha = \frac{1}{1.31}$ and Fig.4 for the resonance case $\alpha = 0$ $(\delta = 0)$.

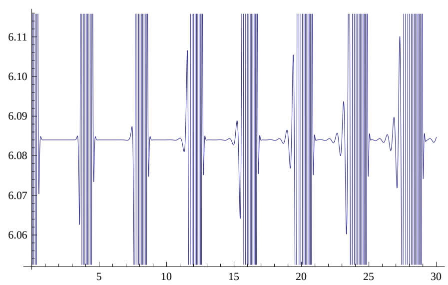

Figure 3: Spin-up JC-spinor population inversion $Z_{+\beta n}(\tau)$, $\tau = gt$: $\quad \alpha = \frac{1}{1.31}$; $\theta = 0$; $\beta = 4$

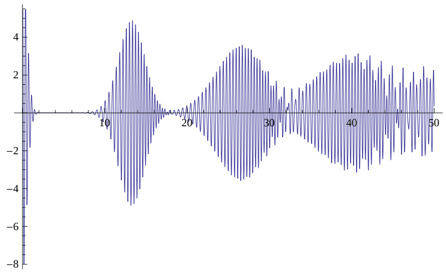

Figure 4: Spin-up JC-spinor population inversion $Z_{+\beta n}(\tau)$, $\tau = gt$: $\alpha = 0$; $\theta = 0$; $\beta = 4$

The phenomenon of quantum collapses and revivals is now enhanced in the dynamics starting with the field mode initially in a coherent state. We notice that the dynamical evolution in the interaction with finite frequency detuning specified by $\alpha \neq 0$ in Fig.3 is characterized by a regular pattern of collapses and revivals over the time duration, while the pattern of collapses and revivals in the resonance dynamics specified by $\alpha = 0$ in Fig.4 is somehow irregular, turning into fractional revivals and then chaotic over long time durations. We observe that at resonance, the atom-field excitation correlation interaction energy vanishes and the dynamical evolution under resonance in Fig.4 describes the standard Jaynes-Cummings interaction commonly characterized by quantum collapses and revivals which develop into fractional revivals and chaotic behavior over long time periods. On the other hand, the consistent regular pattern of collapses and revivals in the non-resonant dynamical evolution in Fig.3 may be attributed to the stability maintained through the atom-field excitation correlation interaction energy in the generalized Jaynes-Cummings interaction in the JC-spinor.

For direct comparison, we have determined the individual atomic spin state population inversion $s_z(t)$ and the field mode photon number $n(t)$ in the general time evolving state $|\Psi(t)\rangle$ in equations $(10d)$, $(10g)$ in explicit form

$$s_z(t) = |u_+|^2 s_z^{+n}(t) + |u_-|^2 s_z^{-n}(t) \ : \qquad s_z^{\pm n}(t) = \pm\frac{1}{2}(\cos^2(\mathcal{R}_{\pm n}t) + (c_{\pm n}^2 - s_{\pm n}^2)\sin^2(\mathcal{R}_{\pm n}t)\ ) \tag{13a}$$

$$n(t) = |u_+|^2 n_{+n}(t) + |u_-|^2 n_{-n}(t) \ : \qquad n_{\pm n}(t) = n \pm s_{\pm n}^2 \sin^2(\mathcal{R}_{\pm n}t) \tag{13b}$$

Comparing equation $(13a)$ with equation $(12d)$ reveals that the JC-spinor and the atomic spin state population inversions $Z_{\pm n}(t)$, $s_z^{\pm n}(t)$, take exactly the same form, which on combining the definitions of $j_{\pm n}$ in equations $(8b)$, $(9b)$, are seen to be directly related according to

$$j_{\pm n} = \frac{1}{2}(n + \frac{1}{2} \pm \frac{1}{2}) \ : \qquad Z_{\pm n}(t) = \left(n + \frac{1}{2} \pm \frac{1}{2}\right) s_z^{\pm n}(t) \tag{13c}$$

It follows immediately that the pattern of time evolution of the atomic spin state population inversion $s_z(t)$ is exactly the same as the time evolution of the JC-spinor state population inversion $Z(t)$ in Fig.1 , Fig.2, agreeing with the results obtained in [12] using the atomic spin reduced density operator in a Jaynes-Cummings interaction.

Setting $u_+ = u_- = \frac{1}{\sqrt{2}}$ in equation $(13b)$, we obtain the general time evolving field mode photon number $n(t)$, which we have plotted in Fig.5 , Fig.6 for initial photon number $n = 1, 16$ with specified parameter value $\alpha = \frac{1}{1.31}$. The regular pattern of dynamical evolution of the field mode photon number reveals quantum collapses and revivals, noting that for initial photon number $n = 16$, Fig.6 agrees precisely with results obtained in a study of the dynamics of the full quantum Rabi model in [13].

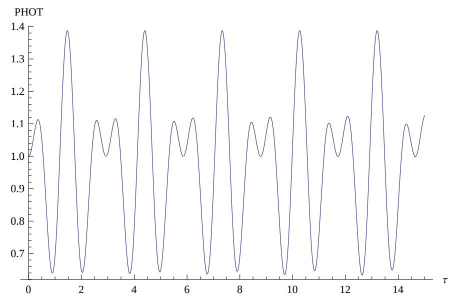

Figure 5: JC-spinor photon number $n(\tau)$, $\tau = gt$: $\quad \alpha = \frac{1}{1.31}$; $\theta = 0$; $n = 1$

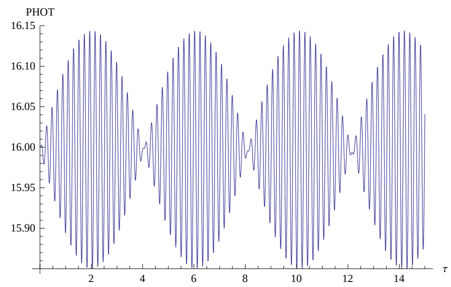

Figure 6: JC-spinor photon number $n(\tau)$, $\tau = gt$: $\quad \alpha = \frac{1}{1.31}$; $\theta = 0$; $n = 16$

Comparing Fig.1 , Fig.5 reveals that the pattern of evolution of the photon number occurs in reverse sense relative to the pattern of evolution of the population inversion, demonstrating the expected alternate photon emission-absorption by the atom and field mode. It is important to note that, in contrast to the population inversion in Fig. 1 , Fig.2 oscillating between positive and negative values, the photon number in Fig.5 , Fig.6 evolves only through positive values. We observe that for large values of the parameter α, the general JC-spinor state population inversion $Z(t)$ turns positive, evolving only through positive values, while the individual atomic spin state population inversion $s_z(t)$ maintains evolution through positive and negative values, but with negative values getting asymptotically close to zero. We have not displayed the large α patterns of evolution here, but the interested reader may easily use the explicit expressions we have determined here to demonstrate these features in Mathematica or any other time evolution plots.

We determine the effect of the JC-spinor state population inversion fluctuations $(\Delta Z(t))^2$ by introducing the signal-to-noise ratio $\Gamma(t)$ defined here by

$$\Gamma(t) = \frac{Z^2(t)}{(\Delta Z(t))^2} \tag{13d}$$

Using the results obtained in equations $(12b)$, $(12d)$, $(12f)$, $(12h)$, we obtain the signal-to-noise ratio $\Gamma(t)$ in explicit form, which by setting $u_+ = u_- = \frac{1}{\sqrt{2}}$, we have plotted against scaled time $\tau = gt$ in Fig.7 , Fig.8 for initial field mode photon numbers $n = 1, 16$ as specified together with the parameter values $\alpha = \frac{1}{1.31}$, $\theta = 0$.

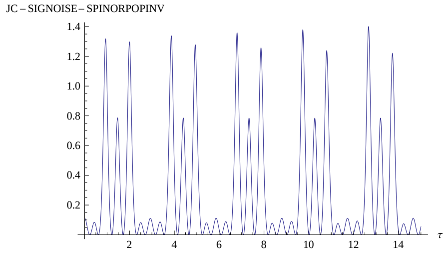

Figure 7: JC-spinor population inversion signal-to-noise ratio $\Gamma(\tau) = Z^2(\tau)/(\Delta Z(\tau))^2$, $\tau = gt$: $\alpha = \frac{1}{1.31}$; $\theta = 0$; $n = 1$

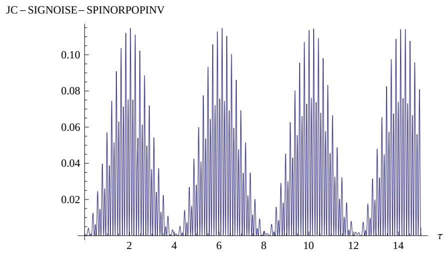

Figure 8: JC-spinor population inversion signal-to-noise ratio $\Gamma(\tau) = Z^2(\tau)/(\Delta Z(\tau))^2$, $\tau = gt$: $\alpha = \frac{1}{1.31}$; $\theta = 0$; $n = 16$

The dynamical evolution is characterized by quantum collapses and revivals signaled by rising and falling peaks with short time oscillatory ripples between the collapses and revivals, clearly evident in the initial single photon ($n = 1$) case in Fig.5. Comparing the peaks in Fig.7 , Fig.8 reveals that in the fast oscillating higher initial photon number $n = 16$ case in Fig.8, the peak of the signal-to-noise ratio decreases with increasing initial field mode photon number, meaning that fluctuations increase with increasing initial field mode photon number.

2.3.3 JC-spinor coherence and coherence fluctuations

The coherence $X(t)$, $Y(t)$ and mean square coherence $\overline{X^2}(t)$, $\overline{Y^2}(t)$ of the JC-spinor in the general time evolving state $|\Psi(t)\rangle$ are determined as the mean values and mean square values of the respective coherence dynamical operators J_x , J_y in equation $(2g)$ according to

$$X(t) = \langle\Psi(t)|J_x|\Psi(t)\rangle \quad ; \quad \overline{X^2}(t) = \langle\Psi(t)|J_x^2|\Psi(t)\rangle$$
$$Y(t) = \langle\Psi(t)|J_y|\Psi(t)\rangle \quad ; \quad \overline{Y^2}(t) = \langle\Psi(t)|J_y^2|\Psi(t)\rangle \qquad (14a)$$

Using $|\Psi_{\pm n}(t)\rangle$ from equation $(10g)$, we substitute $|\Psi(t)\rangle$ from equation $(10d)$ into equation $(14a)$, expand as appropriate and use the $J_\pm$ algebraic relations from equations $(8b)$, $(9b)$ giving

$$J_x|\psi_{\pm n}\rangle = \sqrt{\frac{j_{\pm n}}{2}}\ |\psi_{\mp n\pm 1}\rangle \quad ; \quad J_x|\psi_{\mp n\pm 1}\rangle = \sqrt{\frac{j_{\pm n}}{2}}\ |\psi_{\pm n}\rangle$$
$$J_y|\psi_{\pm n}\rangle = \pm i\sqrt{\frac{j_{\pm n}}{2}}\ |\psi_{\mp n\pm 1}\rangle \quad ; \quad J_y|\psi_{\mp n\pm 1}\rangle = \mp i\sqrt{\frac{j_{\pm n}}{2}}\ |\psi_{\pm n}\rangle \qquad (14b)$$

we apply the $|\psi_\pm\rangle$, $|\psi_{\mp n\pm 1}\rangle$ orthonormalization relations as appropriate to obtain the coherence and mean square coherence in equation $(14a)$ in the final form

$$X(t) = |u_+|^2 X_{+n}(t) + |u_-|^2 X_{-n}(t) \quad ; \quad \overline{X^2}(t) = |u_+|^2\ \overline{X^2_{+n}}(t) + |u_-|^2\ \overline{X^2_{-n}}(t)$$
$$Y(t) = |u_+|^2 Y_{+n}(t) + |u_-|^2 Y_{-n}(t) \quad ; \quad \overline{Y^2}(t) = |u_+|^2\ \overline{Y^2_{+n}}(t) + |u_-|^2\ \overline{Y^2_{-n}}(t) \qquad (14c)$$

where we have identified the spin-up and spin-down JC-spinor coherence $X_{\pm n}(t)$, $Y_{\pm n}(t)$ and mean square coherence $\overline{X^2_{\pm n}}(t)$, $\overline{Y^2_{\pm n}}(t)$ determined in the respective time evolving states $|\Psi_{\pm n}(t)\rangle$ in the form

$$X_{\pm n}(t) = \langle\Psi_{\pm n}(t)|J_x|\Psi_{\pm n}(t)\rangle \quad ; \quad \overline{X^2_{\pm n}}(t) = \langle\Psi_{\pm n}(t)|J_x^2|\Psi_{\pm n}(t)\rangle$$

$$Y_{\pm n}(t) = \langle\Psi_{\pm n}(t)|J_y|\Psi_{\pm n}(t)\rangle \quad ; \quad \overline{Y^2_{\pm n}}(t) = \langle\Psi_{\pm n}(t)|J_y^2|\Psi_{\pm n}(t)\rangle \qquad (14d)$$

The coherence fluctuations in the state $|\Psi(t)\rangle$ are obtained as variances according to

$$(\Delta X(t))^2 = \overline{X^2}(t) - (X(t))^2 \quad ; \quad (\Delta Y(t))^2 = \overline{Y^2}(t) - (Y(t))^2 \qquad (14e)$$

which on substituting the results from equation (14c) and reorganizing as appropriate using the relation $|u_+|^2 + |c_-|^2 = 1$ take the form

$$(\Delta X(t))^2 = |u_+|^2(\Delta X_{+n}(t))^2 + |u_-|^2(\Delta X_{-n}(t))^2 + |u_+|^2|u_-|^2(X_{+n}(t) - X_{-n}(t))^2$$

$$(\Delta Y(t))^2 = |u_+|^2(\Delta Y_{+n}(t))^2 + |u_-|^2(\Delta Y_{-n}(t))^2 + |u_+|^2|u_-|^2(Y_{+n}(t) - Y_{-n}(t))^2 \qquad (14f)$$

where $(\Delta X_{\pm n}(t))^2$, $(\Delta Y_{\pm n}(t))^2$, are the spin-up and spin-down JC-spinor coherence fluctuations obtained as the variances

$$(\Delta X_{\pm n}(t))^2 = \overline{X^2_{\pm n}}(t) - (X_{\pm n}(t))^2 \quad ; \quad (\Delta Y_{\pm n}(t))^2 = \overline{Y^2_{\pm n}}(t) - (Y_{\pm n}(t))^2 \qquad (14g)$$

We now determine the explicit forms of the spin-up and spin-down JC-spinor coherence and coherence fluctuations. Substituting $|\Psi_{\pm n}(t)\rangle$ from equation (10g) into equation (14d), using equation (14b) and applying the appropriate orthonormalization relations, we obtain the spin-up and spin-down JC-spinor coherence and mean square coherence in explicit form

$$X_{\pm n}(t) = \pm\sqrt{\frac{j_{\pm n}}{2}}\; s_{\pm n}\left(\ \sin\theta\sin(2\mathcal{R}_{\pm n}t) + c_{\pm n}\cos\theta(1-\cos(2\mathcal{R}_{\pm n}t))\ \right) \quad ; \quad \overline{X^2_{\pm n}}(t) = \frac{j_{\pm n}}{2}$$

$$Y_{\pm n}(t) = \mp\sqrt{\frac{j_{\pm n}}{2}}\; s_{\pm n}\left(\ \cos\theta\sin(2\mathcal{R}_{\pm n}t) - c_{\pm n}\sin\theta(1-\cos(2\mathcal{R}_{\pm n}t))\ \right) \quad ; \quad \overline{Y^2_{\pm n}}(t) = \frac{j_{\pm n}}{2} \qquad (14h)$$

which we substitute into (14g) to obtain the spin-up and spin-down JC-spinor coherence fluctuations in explicit form

$$(\Delta X_{\pm n}(t))^2 = \frac{j_{\pm n}}{2}\left(\ 1 - s^2_{\pm n}\left(\ \sin\theta\sin(2\mathcal{R}_{\pm n}t) + c_{\pm n}\cos\theta(1-\cos(2\mathcal{R}_{\pm n}t))\ \right)^2\ \right)$$

$$(\Delta Y_{\pm n}(t))^2 = \frac{j_{\pm n}}{2}\left(\ 1 - s^2_{\pm n}\left(\ \cos\theta\sin(2\mathcal{R}_{\pm n}t) - c_{\pm n}\sin\theta(1-\cos(2\mathcal{R}_{\pm n}t))\ \right)^2\ \right) \qquad (14i)$$

Substituting the explicit results from equations (14h) , (14i) into equations (14c) , (14f) as appropriate, we obtain the JC-spinor coherence $X(t)$, $Y(t)$ and coherence fluctuations $(\Delta X(t))^2$, $(\Delta Y(t))^2$ in the general time evolving state $|\Psi(t)\rangle$ in explicit form.

Setting the probability amplitudes $u_+ = u_- = \frac{1}{\sqrt{2}}$, we have plotted the coherence $X(t)$, $Y(t)$ against scaled time $\tau = gt$ in Fig.9 , Fig.10 and Fig.11 , Fig.12, respectively, for photon numbers $n = 1, 16$ as specified together with the parameter values α , $\theta = 0$:

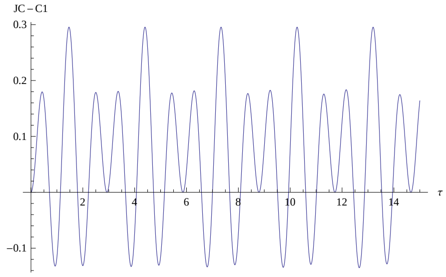

Figure 9: JC-spinor coherence $X(\tau)$, $\tau = gt$: $\alpha = \frac{1}{1.31}$; $\theta = 0$; $n = 1$

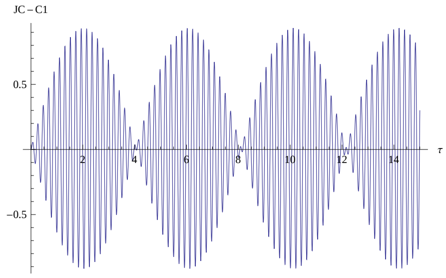

Figure 10: JC-spinor coherence $X(\tau)$, $\tau = gt$: $\alpha = \frac{1}{1.31}$; $\theta = 0$; $n = 16$

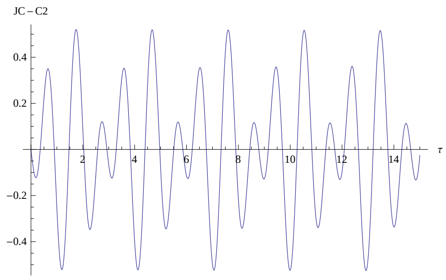

Figure 11: JC-spinor coherence $Y(\tau)$, $\tau = gt$: $\alpha = \frac{1}{1.31}$; $\theta = 0$; $n = 1$

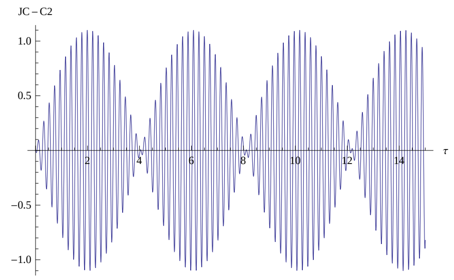

Figure 12: JC-spinor coherence $Y(\tau)$, $\tau = gt$: $\alpha = \frac{1}{1.31}$; $\theta = 0$; $n = 16$

It is clear in Fig.9-Fig.12 that dynamical evolution of the JC-spinor coherence $X(t)$, $Y(t)$ is characterized by a regular pattern of quantum collapses and revivals similar to the evolution profile, but in reverse order relative to the population inversion in Fig.1 , Fig.2. An interesting feature which we have not displayed in a plot is that the x-coherence $X(t)$ vanishes at resonance $\omega = \omega_0$ ($\alpha = 0$), while y-coherence $Y(t)$ and the population inversion $Z(t)$ remain finite.

Coherence fluctuations are useful in determining the quantum nature of a system. For the JC-spinor, simultaneous measurement of the coherence $X(t)$, $Y(t)$ in the general time evolving quantum state $|\Psi(t)\rangle$ is governed by the Heisenberg uncertainty principle, which according to the respective coherence operator J_x , J_y commutation relation $[J_x \ , \ J_y] = iJ_z$, takes the final form

$$(\Delta X(t))^2(\Delta Y(t))^2 \geq \frac{1}{4}Z^2(t) \qquad (14j)$$

where the coherence fluctuations $(\Delta X(t))^2$, $(\Delta Y(t))^2$ and the state population inversion $Z(t)$ have been determined in explicit form above. Setting $u_+ = u_- = \frac{1}{\sqrt{2}}$ as appropriate in each case, we demonstrate the Heisenberg uncertainty principle in a common plot of the uncertainty product $(\Delta X(\tau))^2(\Delta Y(\tau))^2$ and population inversion square $\frac{1}{4}Z^2(\tau)$ against scaled time $\tau = gt$ in Fig.13 , Fig.14 for initial field mode photon numbers $n = 1, 16$ as specified together with the parameter values α , $\theta = 0$:

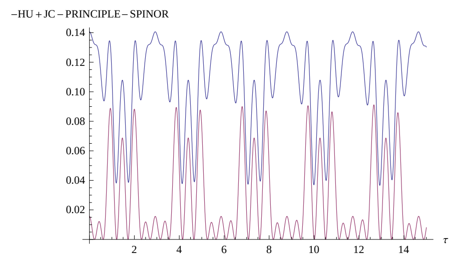

Figure 13: JC-spinor Heisenberg uncertainty principle $(\Delta X(\tau))^2(\Delta Y(\tau))^2 \geq \frac{1}{4}Z^2(\tau)$, $\tau = gt$: $\alpha = \frac{1}{1.31}$; $\theta = 0$; $n = 1$

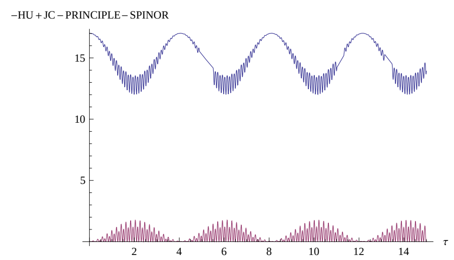

Figure 14: JC-spinor Heisenberg uncertainty principle $(\Delta X(\tau))^2(\Delta Y(\tau))^2 \geq \frac{1}{4}Z^2(\tau)$, $\tau = gt$: $\alpha = \frac{1}{1.31}$; θ ; $n = 16$

We observe that the time evolution of the uncertainty product $(\Delta X(\tau))^2(\Delta Y(\tau))^2$ plotted in blue and the state population inversion square $\frac{1}{4}Z^2(\tau)$ plotted in red each follows a regular pattern characterized by quantum collapses and revivals. It is clear in Fig.13 , Fig.14 that over the duration of time evolution, the corresponding profiles in each pattern essentially fit into each other and the uncertainty product values (blue) are always larger or equal to the corresponding population inversion square values (red), precisely in agreement with the Heisenberg uncertainty relation in equation ($14j$). In particular, for initial field mode photon number $n = 1$ case in Fig.13, the peaks of the uncertainty product are always higher than the corresponding peaks of the population inversion square, but the lower values of the uncertainty product coincide precisely with the upper values on the peaks of the population inversion square, thus accounting for the equality in equation ($14j$), while for the higher initial field mode photon number $n = 16$ case in Fig.14 where the evolution takes reverse order, the values of the uncertainty product are much larger than the corresponding values of the population inversion square, meaning that for such large initial field mode photon numbers, the Heisenberg uncertainty principle is strictly characterized by the inequality in equation ($14j$), thus agreeing with the earlier interpretation that fluctuations increase with increasing initial field mode photon number in the dynamical evolution of the JC-spinor.

Up to this stage, we have provided the algebraic structure and the basic dynamical features of the JC-spinor. We proceed to complete the development of the spinor interpretation of quasiparticle excitations generated in the quantum Rabi model of light-matter interactions by introducing the appropriate dynamical operators to determine the algebraic structure and dynamical features of spinors generated in the generalized antiJaynes-Cummings interaction mechanism where the atom couples to the anticlockwise rotating component of the quantized field mode, which we refer to as the antiJaynes-Cummings spinors, in short antiJC-spinors.

3 The antiJaynes-Cummings spinor

In this section, we introduce and study the dynamical properties of a composite atom-field quasiparticle excitation generated in an antiJaynes-Cummings interaction mechanism where the atom couples to the anticlockwise rotating field mode component. According to the form of the basic antiJaynes-Cummings Hamiltonian $\overline{H}$ in equation ($1d$), the antiJaynes-Cummings interaction mechanism which generates the quasiparticle excitations is governed by the interaction Hamiltonian component $\overline{H}_I = 2\hbar g(\hat{a}s_- + \hat{a}^\dagger s_+)$. In the physical interpretation we are developing in the present work, we identify the quasiparticle excitation generated in the atom-field antiJaynes-Cummings interaction mechanism as an *antiJaynes-Cummings spinor*, which we shall refer to simply as an antiJC-spinor, thereby distinguishing it from the conventional interpretation of the atom-field quasiparticle excitations as polaritons in standard quantum optics [4-7] or specifically as antipolaritons in [2 , 3].

It follows from the algebraic form of the antiJaynes-Cummings interaction Hamiltonian $\overline{H}_I = 2\hbar g(\hat{a}s_- + \hat{a}^\dagger s_+)$ that the antiJC-spinor may be interpreted as a composite quantized atom-field physical entity characterized by state lowering and raising operators $\mathcal{J}_-$, $\mathcal{J}_+$ obtained as products of the basic atom and field mode state lowering and raising operators $(s_-\ ,\ s_+)$, $(\hat{a}\ ,\ \hat{a}^\dagger)$ in the form

$$\mathcal{J}_- = \hat{a}s_- \qquad ; \qquad \mathcal{J}_+ = \hat{a}^\dagger s_+ \tag{15a}$$

with hermitian coherence dynamical operators $\hat{\overline{\Sigma}}_x$, $\hat{\overline{\Sigma}}_y$ defined in terms of $\mathcal{J}_-$, $\mathcal{J}_+$ in symmetric and antisymmetric forms

$$\hat{\overline{\Sigma}}_x = \mathcal{J}_+ + \mathcal{J}_- \qquad ; \qquad \hat{\overline{\Sigma}}_y = -i(\ \mathcal{J}_+ - \mathcal{J}_-\ ) \tag{15b}$$

Using the basic atom and field mode operator algebraic relations in equation ($2c$), we take normal and antinormal order products of $\mathcal{J}_-$, $\mathcal{J}_+$ to obtain

$$\mathcal{J}_+\mathcal{J}_- = \hat{a}^\dagger\hat{a} - \hat{a}^\dagger\hat{a}s_-s_+ \qquad ; \qquad \mathcal{J}_-\mathcal{J}_+ = s_-s_+ + \hat{a}^\dagger\hat{a}s_-s_+ \tag{15c}$$

which we add and subtract to obtain diagonal symmetric and antisymmetric dynamical operators $\hat{\overline{I}}$, $\hat{\overline{\Sigma}}_z$ in the form

$$\hat{\overline{I}} = \mathcal{J}_+\mathcal{J}_- + \mathcal{J}_-\mathcal{J}_+ \qquad \Rightarrow \qquad \hat{\overline{I}} = \hat{a}^\dagger\hat{a} + s_-s_+$$
$$\hat{\overline{\Sigma}}_z = \mathcal{J}_+\mathcal{J}_- - \mathcal{J}_-\mathcal{J}_+ \qquad \Rightarrow \qquad \hat{\overline{\Sigma}}_z = \hat{a}^\dagger\hat{a} - s_-s_+ - 2\hat{a}^\dagger\hat{a}s_-s_+ \tag{15d}$$

where we interpret $\hat{\overline{I}}$ as the antiJC-spinor identity operator, which we identify as the antipolariton excitation number operator $\hat{\overline{\hat{N}}}$ ($\hat{\overline{I}} = \hat{\overline{\hat{N}}} - 1$) defined earlier in equation ($1f$).

We introduce basic operators $\mathcal{J}_0$, $\mathcal{J}_x$, $\mathcal{J}_y$, $\mathcal{J}_z$ consistent with the definitions of the antiJC-spinor state lowering and raising operators $\mathcal{J}_-$, $\mathcal{J}_+$ in the form

$$\mathcal{J}_0 = \frac{1}{2}\,\hat{\bar{I}} \qquad ; \qquad \mathcal{J}_x = \frac{1}{2}\,\hat{\bar{\Sigma}}_x \qquad ; \qquad \mathcal{J}_y = \frac{1}{2}\,\hat{\bar{\Sigma}}_y \qquad ; \qquad \mathcal{J}_z = \frac{1}{2}\,\hat{\bar{\Sigma}}_z \tag{15e}$$

which on using equations (15b) , (15d) take suitable forms

$$\mathcal{J}_0 = \frac{1}{2}(\mathcal{J}_+\mathcal{J}_- + \mathcal{J}_-\mathcal{J}_+) \; ; \quad \mathcal{J}_z = \frac{1}{2}(\mathcal{J}_+\mathcal{J}_- - \mathcal{J}_-\mathcal{J}_+) \quad \Rightarrow \quad \mathcal{J}_+\mathcal{J}_- = \mathcal{J}_0 + \mathcal{J}_z \; ; \quad \mathcal{J}_-\mathcal{J}_+ = \mathcal{J}_0 - \mathcal{J}_z$$

$$\mathcal{J}_x = \frac{1}{2}(\mathcal{J}_+ + \mathcal{J}_-) \quad ; \quad \mathcal{J}_y = -\frac{i}{2}(\mathcal{J}_+ - \mathcal{J}_-) \tag{15f}$$

where in defining $\mathcal{J}_0$, $\mathcal{J}_z$, we take note of the explicit forms in equation (15d).

Using standard algebraic property of atomic spin state lowering and raising operators s_- , s_+ given in equations (3a) , (5a), we obtain the corresponding algebraic property of the antiJC-spinor state lowering and raising operators $\mathcal{J}_-$, $\mathcal{J}_+$ in the form

$$\mathcal{J}_-^2 = 0 \qquad ; \qquad \mathcal{J}_+^2 = 0 \tag{16a}$$

which we apply to determine the following algebraic relations for the quadratic operators $\mathcal{J}_x^2$, $\mathcal{J}_y^2$, $\mathcal{J}_z^2$ with respect to $\mathcal{J}_0$ using the definitions in equation (15f) in the form

$$\mathcal{J}_x^2 = \frac{1}{2}\mathcal{J}_0 \qquad ; \qquad \mathcal{J}_y^2 = \frac{1}{2}\mathcal{J}_0 \qquad ; \qquad \mathcal{J}_z^2 = \mathcal{J}_0^2 \tag{16b}$$

We introduce the antiJC-spinor Casimir operator $\mathcal{J}^2$ defined in standard form by

$$\mathcal{J}^2 = \mathcal{J}_x^2 + \mathcal{J}_y^2 + \mathcal{J}_z^2 \tag{16c}$$

which on substituting $\mathcal{J}_x$, $\mathcal{J}_y$, $\mathcal{J}_0$ from equation (15f) or using the quadratic operator algebraic relations obtained in equation (16b) takes the form

$$\mathcal{J}^2 = \frac{1}{2}(\mathcal{J}_+\mathcal{J}_- + \mathcal{J}_-\mathcal{J}_+) + \mathcal{J}_z^2 = \mathcal{J}_0 + \mathcal{J}_z^2 \qquad \Rightarrow \quad \mathcal{J}^2 = \mathcal{J}_0(\mathcal{J}_0 + 1) \tag{16d}$$

The definition of $\mathcal{J}_z$ in equation (15f) easily provides an important algebraic relation

$$[\ \mathcal{J}_+ \ , \ \mathcal{J}_- \ ] = 2\mathcal{J}_z \tag{17a}$$

Here again, we observe that this commutation bracket relation of the state lowering and raising operators $\mathcal{J}_\pm$, compared to the corresponding atomic spin commutation bracket $[s_+ \ , \ s_-] = 2s_z$ where s_z is the atomic spin state population inversion operator, leads us to interpret the dynamical operator $\mathcal{J}_z$ defined explicitly by equations (15d) , (15e) as the antiJC-spinor state population inversion operator, which will be established through its algebraic operations on coupled qubit state vectors in subsection 3.2 below.

Using the definitions of $\mathcal{J}_0$, $\mathcal{J}_z$, $\mathcal{J}_x$, $\mathcal{J}_y$, $\mathcal{J}^2$ in equations (15f) , (16d), we apply the algebraic property of $\mathcal{J}_-$, $\mathcal{J}_+$ in equation (16a) to obtain the following closed algebraic relations, including equation (17a):

$$[\mathcal{J}_0 \ , \ \mathcal{J}_-] = 0 \quad ; \quad [\mathcal{J}_0 \ , \ \mathcal{J}_+] = 0 \quad ; \quad [\mathcal{J}_0 \ , \ \mathcal{J}_x] = 0 \quad ; \quad [\mathcal{J}_0 \ , \ \mathcal{J}_y] = 0 \quad ; \quad [\mathcal{J}_0 \ , \ \mathcal{J}_z] = 0 \tag{17b}$$

$$\{\mathcal{J}_z \ , \ \mathcal{J}_-\} = 0 \quad ; \quad \{\mathcal{J}_z \ , \ \mathcal{J}_+\} = 0 \quad ; \quad \{\mathcal{J}_z \ , \ \mathcal{J}_x\} = 0 \quad ; \quad \{\mathcal{J}_z \ , \ \mathcal{J}_y\} = 0 \quad ; \quad \{\mathcal{J}_x \ , \ \mathcal{J}_y\} = 0 \tag{17c}$$

$$[\mathcal{J}^2 \ , \ \mathcal{J}_-] = 0 \; ; \quad [\mathcal{J}^2 \ , \ \mathcal{J}_+] = 0 \; ; \quad [\mathcal{J}^2 \ , \ \mathcal{J}_0] \; ; \quad [\mathcal{J}^2 \ , \ \mathcal{J}_x] = 0 \; ; \quad [\mathcal{J}^2 \ , \ \mathcal{J}_y] = 0 \; ; \quad [\mathcal{J}^2 \ , \ \mathcal{J}_z] = 0 \tag{17d}$$

$$[\mathcal{J}_+ \ , \ \mathcal{J}_-] = 2\mathcal{J}_z \qquad ; \qquad [\mathcal{J}_z \ , \ \mathcal{J}_+] = \mathcal{J}_+\hat{\bar{I}} \qquad ; \qquad [\mathcal{J}_z \ , \ \mathcal{J}_-] = -\mathcal{J}_-\hat{\bar{I}}$$

$$[\mathcal{J}_x \ , \ \mathcal{J}_y] = i\mathcal{J}_z \qquad ; \qquad [\mathcal{J}_y \ , \ \mathcal{J}_z] = i\mathcal{J}_x\hat{\bar{I}} \qquad ; \qquad [\mathcal{J}_z \ , \ \mathcal{J}_x] = i\mathcal{J}_y\hat{\bar{I}} \tag{17e}$$

where $\hat{\bar{I}} = 2\mathcal{J}_0$ in equation (17e) is the antiJC-spinor identity (or conserved excitation number) operator, which commutes with $\mathcal{J}_\pm$, $\mathcal{J}_x$, $\mathcal{J}_y$. Here again, we note that the evaluation of equations (17b)-(17d) using equation $(16a)$ is straightforward, while equation $(17e)$ includes a step applying equation $(15f)$ according to the procedure presented in the JC-spinor case in equation $(4f)$. The results in equation $(17d)$ establish the standard algebraic property of a Casimir operator commuting with all the operators which generate the closed algebra of a symmetry group. The full set of closed algebraic relations in equations $(16a)$-$(16d)$, $(17a)$-$(17e)$ constitute a closed generalized $SU(2)$ Lie algebra and the associated anticommutation relations of a spinor characterized by dynamical operators $\mathcal{J}_0$, $\mathcal{J}_z$, $\mathcal{J}_\pm$, $\mathcal{J}_x$, $\mathcal{J}_y$ and a Casimir operator $\mathcal{J}^2$, precisely similar to the closed $SU(2)$ Lie algebra and the associated anticommutation relations generated by the basic spin operators of a two-state atomic spin or spin-$\frac{1}{2}$ particle presented in equations $(5a)$-$(5d)$. We identify this spinor as the antiJC-spinor, which arises as an atom-field quasiparticle excitation in an antiJaynes-Cummings interaction mechanism. The antiJC-spinor dynamical operators are interpreted as generators of a generalized $SU(2)$ Lie algebra.

3.1 The antiJC-spinor Hamiltonian

To determine the algebraic form of the antiJC-spinor Hamiltonian, we consider that the conserved excitation number operator is defined in symmetrized form in terms of normal and antinormal order products of the lowering and raising operators $\mathcal{J}_-$, $\mathcal{J}_+$, which constitute the identity and state population inversion operators $\mathcal{J}_0$, $\mathcal{J}_z$ according to equations $(15c)$-$(15f)$. It follows from equation $(15c)$ that in the atom-field antiJaynes-Cummings interaction mechanism which forms an antiJC-spinor, the normal order form $\mathcal{J}_+\mathcal{J}_- = \hat{a}^\dagger\hat{a} - \hat{a}^\dagger\hat{a}s_-s_+$ provides the effective field mode excitation number operator, while the antinormal order form $\mathcal{J}_-\mathcal{J}_+ = s_-s_+ + \hat{a}^\dagger\hat{a}s_-s_+$ provides the effective atomic spin excitation number operator, each modified by the atom-field excitation number correlation operator $\mp\hat{a}^\dagger\hat{a}s_-s_+$ arising from the atom-field mode coupling as appropriate. Noting the relation $s_-s_+ = s_0 - s_z$, the effective free evolution Hamiltonian of an antiJC-spinor is then obtained as the difference of the effective field mode component $\hbar\omega\mathcal{J}_+\mathcal{J}_-$ and the atomic spin component $\hbar\omega_0\mathcal{J}_-\mathcal{J}_+$ in the form $\hbar\omega\mathcal{J}_+\mathcal{J}_- - \hbar\omega_0\mathcal{J}_-\mathcal{J}_+$, while the interaction Hamiltonian is obtained in the general form as a linear combination of the coherence components in the form $\overline{a}_x\hat{\overline{\Sigma}}_x + a_y\hat{\overline{\Sigma}}_y$. We therefore define the antiJC-spinor Hamiltonian $\overline{\mathcal{H}}$ in terms of the dynamical operators in the general form

$$\overline{\mathcal{H}} = \hbar\omega\mathcal{J}_+\mathcal{J}_- - \hbar\omega_0\mathcal{J}_-\mathcal{J}_+ + \hbar(\overline{a}_x\hat{\overline{\Sigma}}_x + \overline{a}_y\hat{\overline{\Sigma}}_y) \tag{18a}$$

where ω , ω_0 are the field mode and atom angular frequencies for the free evolution energy of composite atom-field system, while $\overline{a}_x$, $\overline{a}_y$ are real physical parameters defining the atom-field interaction which forms the antiJC-spinor.

Substituting $\hat{\overline{\Sigma}}_x = \mathcal{J}_+ + \mathcal{J}_-$, $\hat{\overline{\Sigma}}_y = -i(\mathcal{J}_+ - \mathcal{J}_-)$ from equation $(15b)$ into equation $(18a)$ and symmetrizing the first two terms according to $2(aA - bB) = (a+b)(A-B) + (a-b)(A+B)$ then substituting $\mathcal{J}_0$, $\mathcal{J}_z$ as defined in equations $(15d)$, $(15f)$ provides the antiJC-spinor Hamiltonian in the form

$$\overline{\mathcal{H}} = -\hbar\delta\mathcal{J}_0 + \hbar\overline{\delta}\mathcal{J}_z + 2\hbar g(e^{-i\overline{\theta}}\mathcal{J}_+ + e^{i\overline{\theta}}\mathcal{J}_-) \quad ; \quad \delta = \omega_0 - \omega \quad ; \quad \overline{\delta} = \omega_0 + \omega \tag{18b}$$

where we have introduced blue and red sideband frequency detunings $\overline{\delta} = \omega_0 + \omega$, $\delta = \omega_0 - \omega$ as defined earkier in equation $(6b)$ and we have redefined interaction parameters to coincide with the parameter definitions in the antiJaynes-Cummings Hamiltonian $\overline{H}$ in equation $(1d)$ for ease of comparison in the form

$$\overline{a}_x \mp i\overline{a}_y = |\overline{a}_x \mp i\overline{a}_y|e^{\mp i\overline{\theta}} \quad ; \quad |\overline{a}_x \mp i\overline{a}_y| = 2g \quad ; \quad \tan\overline{\theta} = \frac{\overline{a}_y}{\overline{a}_x} \tag{18c}$$

It follows from the commutation relations in equations $(17b)$, $(17d)$ that the operators $\mathcal{J}_0$, $\mathcal{J}^2$ commute with the Hamiltonian $\overline{\mathcal{H}}$ according to

$$[\ \mathcal{J}_0\ ,\ \overline{\mathcal{H}}\ ] = 0 \quad ; \quad [\ \mathcal{J}^2\ ,\ \overline{\mathcal{H}}\ ] = 0 \tag{18d}$$

meaning that $\mathcal{J}_0$, $\mathcal{J}^2$ are conserved dynamical operators of the antiJC-spinor. These operators can be used to determine the entangled state vectors which specify the quantum state space of the antiJC-spinor. In particular, the emergence of the Casimir operator $\mathcal{J}^2$ and the state population inversion operator $\mathcal{J}_z$ determined algebraically and defined in equations $(15c)$-$(15f)$, now provides a standard approach for determining antiJC-spinor state eigenvectors and eigenvalues through the $SU(2)$ Lie algebra.

3.1.1 Generalized antiJaynes-Cummings model : atom-field excitation correlation energy

We now determine the physical nature of the dynamics generated by the antiJC-spinor Hamiltonian $\overline{\mathcal{H}}$. Substituting $\mathcal{J}_+\mathcal{J}_-$, $\mathcal{J}_-\mathcal{J}_+$ from equation (15c) into equation (18a), setting $\overline{a}_y = 0$, $\overline{a}_x = 2g$ and introducing the definition of $\mathcal{J}_\mp$ from equation (15a), we obtain the Hamiltonian in the form

$$a_y = 0 \quad ; \quad a_x = 2g \ : \qquad \overline{\mathcal{H}} = \hbar\omega(\hat{a}^\dagger\hat{a} - \hat{a}^\dagger\hat{a}s_-s_+) - \hbar\omega_0(s_-s_+ + \hat{a}^\dagger\hat{a}s_-s_+) + 2\hbar g(\hat{a}s_- + \hat{a}^\dagger s_+) \tag{19a}$$

which on introducing the relations $\hat{a}^\dagger\hat{a} = \hat{a}\hat{a}^\dagger - 1$, $-s_-s_+ = s_z - \frac{1}{2}$ in the free field and atom components is easily reorganized in the form

$$\overline{\mathcal{H}} = \hbar\omega\hat{a}\hat{a}^\dagger + \hbar\omega_0 s_z + 2\hbar g(\hat{a}s_- + \hat{a}^\dagger s_+) - \frac{1}{2}\hbar\omega_0 - \hbar\omega - \hbar(\omega_0 + \omega)\hat{a}^\dagger\hat{a}s_-s_+$$

$$\Rightarrow \quad \overline{\mathcal{H}} = \overline{H} - \hbar(\omega_0 + \omega)\hat{a}^\dagger\hat{a}s_-s_+ \tag{19b}$$

where $\overline{H}$ is the standard antiJaynes-Cummings Hamiltonian in equation (1d). It is clear in equation (19b) that the antiJC-spinor Hamiltonian $\overline{\mathcal{H}}$ is a generalization of the basic antiJaynes-Cummings Hamiltonian to include an atom-field excitation number correlation operator component $-\hbar(\omega_0+\omega)\hat{a}^\dagger\hat{a}s_-s_+$, which generates an atom-field excitation correlation energy. Notice that in the antiJC-spinor case, the atom-field excitation correlation energy depends on the blue-sideband detuning frequency $\overline{\delta} = \omega_0 + \omega$ which does not vanish at the familiar resonance frequency $\omega = \omega_0$, in contrast to the JC-spinor case where the atom-field excitation correlation energy depends on the red-sideband detuning frequency $\delta = \omega_0 - \omega$ and therefore vanishes under resonance conditions as explained earlier. We establish below that the general dynamics generated by the antiJC-spinor Hamiltonian $\overline{\mathcal{H}}$, which we now identify as a generalized antiJaynes-Cummings Hamiltonian, is characterized by alternate field and atom blue or red frequency-shifts associated with the excitation number correlation energy.

Here again, we emphasize that the atom-field excitation number correlation operator component $-\hbar(\omega_0 + \omega)\hat{a}^\dagger\hat{a}s_-s_+$ which extends the basic antiJaynes-Cummings Hamiltonian $\overline{H}$ to the generalized form $\overline{\mathcal{H}}$ of the antiJC-spinor in equation (19b) is not externally introduced, but arises naturally from the algebraic property of normal and antinormal operator ordering of the products of the antiJC-spinor state lowering and raising operators $\mathcal{J}_-$, $\mathcal{J}_+$ taken in equation (15c) to define the identity and state population inversion operators $\mathcal{J}_0$, $\mathcal{J}_z$ according to equations (15d)-(15f). We re-emphasize the interpretation that the atom-field excitation number correlation operator, which modifies the atom and field mode excitation number operators according to equation (15c), is an intrinsic dynamical property of the algebraic structure of the atom-field interaction mechanism, which does not directly manifest itself externally, thus generally missing in standard atom-field interaction models as explained earlier.

3.2 The antiJC-spinor quantum state space

Having determined all the dynamical operators, including the Casimir operator and Hamiltonian, we now complete the characterization of the internal structure of the antiJC-spinor by determining the state vectors and energy spectrum which specify the quantum state space of the system. As we explained earlier in section 1, here again we achieve clarity by developing the spin-up and spin-down antiJC-spinor quantum state spaces separately in subsections 3.2.1 and 3.2.2 below. We present the basic algebraic operations of the antiJC-spinor dynamical operators on the initial n-photon spin-up and spin-down state vectors $|\psi_{\pm n}\rangle$, which generate the respective coupled qubit state vectors, state eigenvectors and eigenvalues. The spin-up and spin-down antiJC-spinor state energy eigenvalues determined explicitly reveal alternate atom and field mode blue or red frequency-shifts in the upper or lower spectrum.

3.2.1 Spin-up antiJC-spinor

The initial state vector of the spin-up antiJC-spinor is the n-photon spin-up state vector $|\psi_{+n}\rangle$ defined in equation (1h). Applying the antiJC-spinor state lowering and raising operators $\mathcal{J}_\mp$ defined in equation (15a) on the initial state vector $|\psi_{+n}\rangle$ in equation (1h), using equation (1e) and introducing the corresponding transition state vector $|\psi_{-n-1}\rangle$ to define the coupled pair of initial spin-up antiJC-spinor qubit state vectors in the form

$$|\psi_{+n}\rangle = |+n\rangle \qquad ; \qquad |\psi_{-n-1}\rangle = |-n-1\rangle \tag{20a}$$

we obtain the coupled initial qubit state algebraic operations

$$\mathcal{J}_-|\psi_{+n}\rangle = \sqrt{2j_{-n}}|\psi_{-n-1}\rangle \quad ; \quad \mathcal{J}_+|\psi_{-n-1}\rangle = \sqrt{2j_{-n}}|\psi_{+n}\rangle \quad ; \quad \mathcal{J}_+|\psi_{+n}\rangle = 0 \quad ; \quad \mathcal{J}_-|\psi_{-n-1}\rangle = 0$$

$$j_{-n} = \frac{n}{2} \quad ; \quad n = 0, 1, 2, 3, ... \tag{20b}$$

where we have introduced the spin-up antiJC-spinor quantum number j_{-n}, which according to the definition given earlier in equation (9b), coincides with the spin-down JC-spinor quantum number.

The dynamical operators $\mathcal{J}_0$, $\mathcal{J}_z$ and the Casimir operator $\mathcal{J}^2$ from equations (15f) , (16d) generate eigenvalue equations on $|\psi_{+n}\rangle$, $|\psi_{-n-1}\rangle$ in the form

$$\mathcal{J}_0|\psi_{+n}\rangle = j_{-n}|\psi_{+n}\rangle \quad ; \quad \mathcal{J}_0|\psi_{-n-1}\rangle = j_{-n}|\psi_{-n-1}\rangle$$

$$\mathcal{J}_z|\psi_{+n}\rangle = j_{-n}|\psi_{+n}\rangle \quad ; \quad \mathcal{J}_z|\psi_{-n-1}\rangle = -j_{-n}|\psi_{-n-1}\rangle$$

$$\mathcal{J}^2|\psi_{+n}\rangle = j_{-n}(j_{-n}+1)|\psi_{+n}\rangle \quad ; \quad \mathcal{J}^2|\psi_{-n-1}\rangle = j_{-n}(j_{-n}+1)|\psi_{-n-1}\rangle \tag{20c}$$

The physical interpretation of $\mathcal{J}_z$ as the antiJC-spinor state population inversion operator is clearly demonstrated in the eigenvalue equations it generates on the coupled initial qubit state vectors $|\psi_{+n}\rangle$, $|\psi_{-n-1}\rangle$ in equation (20c), with corresponding upper or lower eigenvalues $\pm j_{-n}$, such that we may interpret $|\psi_{+n}\rangle$ as the upper and $|\psi_{-n-1}\rangle$ as the lower initial qubit states of the spin-up antiJC-spinor. Here again, we note that the eigenvalue equations generated by the antiJC-spinor Casimir operator $\mathcal{J}^2$ take precisely the form of the standard quantum mechanical total angular momentum Casimir operator eigenvalue equations with quantum number j_{-n} taking both integer and half-integer values according to the definition in equation (20b).

3.2.2 Spin-down antiJC-spinor

The initial state vector of the spin-down antiJC-spinor is the n-photon spin-down state vector $|\psi_{-n}\rangle$ defined in equation (1h). Applying the antiJC-spinor state lowering and raising operators $\mathcal{J}_\pm$ defined in equation (15a) on the initial state vector $|\psi_{-n}\rangle$ in equation (1h), using equation (1e) and introducing the corresponding transition state vector $|\psi_{+n+1}\rangle$ to define the coupled pair of initial spin-down antiJC-spinor qubit state vectors in the form

$$|\psi_{-n}\rangle = |-n\rangle \quad ; \quad |\psi_{+n+1}\rangle = |+n+1\rangle \tag{21a}$$

we obtain the coupled initial qubit state algebraic operations

$$\mathcal{J}_+|\psi_{-n}\rangle = \sqrt{2j_{+n}}|\psi_{+n+1}\rangle \quad ; \quad \mathcal{J}_-|\psi_{+n+1}\rangle = \sqrt{2j_{+n}}|\psi_{-n}\rangle \quad ; \quad \mathcal{J}_-|\psi_{-n}\rangle = 0 \quad ; \quad \mathcal{J}_+|\psi_{+n+1}\rangle = 0$$

$$j_{+n} = \frac{(n+1)}{2} \quad ; \quad n = 0, 1, 2, 3, ... \tag{21b}$$

where we have introduced the spin-down antiJC-spinor quantum number j_{+n}, which according to the definition given earlier in equation (8b), coincides with the spin-up JC-spinor quantum number.

The dynamical operators $\mathcal{J}_0$, $\mathcal{J}_z$ and the Casimir operator $\mathcal{J}^2$ from equations (15f) , (16d) generate eigenvalue equations on $|\psi_{-n}\rangle$, $|\psi_{+n+1}\rangle$ in the form

$$\mathcal{J}_0|\psi_{-n}\rangle = j_{+n}|\psi_{-n}\rangle \quad ; \quad \mathcal{J}_0|\psi_{+n+1}\rangle = j_{+n}|\psi_{+n+1}\rangle$$

$$\mathcal{J}_z|\psi_{-n}\rangle = -j_{+n}|\psi_{-n}\rangle \quad ; \quad \mathcal{J}_z|\psi_{+n+1}\rangle = j_{+n}|\psi_{+n+1}\rangle$$

$$\mathcal{J}^2|\psi_{-n}\rangle = j_{+n}(j_{+n}+1)|\psi_{-n}\rangle \quad ; \quad \mathcal{J}^2|\psi_{-n+1}\rangle = j_{+n}(j_{+n}+1)|\psi_{+n+1}\rangle \tag{21c}$$

Here again, the physical interpretation of $\mathcal{J}_z$ as the antiJC-spinor state population inversion operator is clearly demonstrated in the eigenvalue equations it generates on the coupled initial qubit state vectors $|\psi_{-n}\rangle$, $|\psi_{+n+1}\rangle$ in equation (21c), with corresponding lower or upper eigenvalues $\mp j_{+n}$, such that we may interpret $|\psi_{-n}\rangle$ as the lower and $|\psi_{+n+1}\rangle$ as the upper initial qubit states of the spin-down antiJC-spinor. The eigenvalue equations generated by the antiJC-spinor Casimir operator $\mathcal{J}^2$ take precisely the form of the standard quantum mechanical total angular momentum Casimir operator eigenvalue equations with quantum number j_{+n} taking both integer and half-integer values according to the definition in equation (21b).

The basic qubit state vectors and algebraic operations in equations (20a)-(20c), (21a)-(21c), together with the algebraic properties in equations (16a)-(16d) , (17a)-(17e), can be used to determine all the dynamical properties of the antiJC-spinor, which we now present in subsection 3.3 below.

3.3 Dynamical evolution of the antiJC-spinor

We now determine the internal dynamics of the antiJC-spinor generated by the Hamiltonian $\overline{\mathcal{H}}$ in equation (18a), reorganized in the appropriate form in equation (18b). It is evident in subsections 3.2.1 , 3.2.2 that the complete quantum state space generated by the antiJC-spinor Hamiltonian $\overline{\mathcal{H}}$ is composed of two orthogonal and independent quantum spaces, depending on the initial spin-up or spin-down state of the atom, namely, the spin-up antiJC-spinor quantum state space specified by the coupled pair of initial qubit state vectors $(|\psi_{+n}\rangle\ ,\ |\psi_{-n-1}\rangle)$ and the spin-down antiJC-spinor quantum state space specified by the coupled pair of initial qubit state vectors $(|\psi_{-n}\rangle\ ,\ |\psi_{+n+1}\rangle)$, which are orthogonal according to the relations

$$\langle\psi_{+n}|\psi_{-n}\rangle = 0 \quad ; \quad \langle\psi_{+n}|\psi_{+n+1}\rangle = 0 \quad ; \quad \langle\psi_{-n-1}|\psi_{-n}\rangle = 0 \quad ; \quad \langle\psi_{-n-1}|\psi_{+n+1}\rangle = 0 \qquad (22a)$$

Hence, instead of studying the internal dynamics of spin-up and spin-down antiJC-spinors separately, we find it more effective to consider the basic antiJC-spinor formed in initial superposition state $|\psi\rangle$ of the n-photon spin-up and spin-down states $|\psi_{+n}\rangle$, $|\psi_{-n}\rangle$ as defined in equation (1j).

Dynamical evolution of the antiJC-spinor described by the general time evolving state vector $|\overline{\Psi}(t)\rangle$ is governed by time evolution operator $\overline{\mathcal{U}}(t)$ generated by the Hamiltonian $\overline{\mathcal{H}}$ through the time-dependent Schroedinger equation as presented in equation (10b), but now substituting $|\overline{\Psi}(t)\rangle$, $\overline{\mathcal{H}}$ for the antiJC-spinor. Here again, we note that the interaction parameters $\overline{a}_x$, $\overline{a}_y$ which characterize the Hamiltonian in equation (18a) may be time-dependent or time-independent, but we choose the time-independent case and consider the Hamiltonian $\overline{\mathcal{H}}$ to be time-independent, postponing the time-dependent form for specified cases.

For time-independent Hamiltonian $\overline{\mathcal{H}}$ in equation (18b), the general time evolving state vector $|\ \overline{\Psi}(t)\rangle$ of the antiJC-spinor initially in the superposition state $|\psi\rangle$ is easily obtained through a simple integration of the time-dependent Schroedinger equation in the form

$$|\overline{\Psi}(t)\rangle = \overline{\mathcal{U}}(t)|\psi\rangle \qquad ; \qquad \overline{\mathcal{U}}(t) = e^{-\frac{i}{\hbar}\overline{\mathcal{H}}t} \qquad \Rightarrow \qquad \overline{\mathcal{U}}(t) = e^{-2igt(\ \overline{\alpha}\mathcal{J}_z + e^{-i\overline{\theta}}\mathcal{J}_+ + e^{i\overline{\theta}}\mathcal{J}_-)}\ e^{i\delta t\mathcal{J}_0} \qquad (22b)$$

where we have applied the commutation property $[\ \mathcal{J}_0\ ,\ \overline{\alpha}\mathcal{J}_z + e^{-i\overline{\theta}}\mathcal{J}_+ + e^{i\overline{\theta}}\mathcal{J}_-\ ] = 0$ to factorize the time evolution operator $\overline{\mathcal{U}}(t)$ in appropriate form for ease of evaluation. Substituting the initial state vector $|\psi\rangle$ from equation (1j) into equation (22b) provides $|\ \overline{\Psi}(t)\rangle$ as a superposition of the time evolving spin-up and spin-down antiJC-spinor state vectors $|\ \overline{\Psi}_{+n}(t)\rangle$, $|\ \overline{\Psi}_{-n}(t)\rangle$, respectively, in the form

$$|\ \overline{\Psi}(t)\rangle = u_+|\ \overline{\Psi}_{+n}(t)\rangle + u_-|\ \overline{\Psi}_{-n}(t)\rangle \qquad ; \qquad |\ \overline{\Psi}_{+n}(t)\rangle = \overline{\mathcal{U}}(t)|\psi_{+n}\rangle \quad ; \quad |\ \overline{\Psi}_{-n}(t)\rangle = \overline{\mathcal{U}}(t)|\psi_{-n}\rangle \qquad (22c)$$

Substituting $\overline{\mathcal{U}}(t)$ from equation (22b) into equation (22c) and applying the $\mathcal{J}_0$ eigenvalue equations from (20c) , (21c), we express the time evolving spin-up and spin-down antiJC-spinor state vectors $|\ \overline{\Psi}_{+n}(t)\rangle$, $|\ \overline{\Psi}_{-n}(t)\rangle$ in the form

$$|\ \overline{\Psi}_{+n}(t)\rangle = e^{i\delta j_{-n}t}e^{-2igt\ (\overline{\alpha}\mathcal{J}_z + e^{-i\overline{\theta}}\mathcal{J}_+ + e^{i\overline{\theta}}\mathcal{J}_-)}\ |\psi_{+n}\rangle$$

$$|\ \overline{\Psi}_{-n}(t)\rangle = e^{i\delta j_{+n}t}e^{-2igt\ (\overline{\alpha}\mathcal{J}_z + e^{-i\overline{\theta}}\mathcal{J}_+ + e^{i\overline{\theta}}\mathcal{J}_-)}\ |\psi_{-n}\rangle \qquad (22d)$$

Expanding the interaction time evolution operator in even and odd power terms, writing $(-i)^{2k} = (-1)^k$, $(-i)^{2k+1} = -i(-1)^k$ and using the algebraic relations from equations (16a) , (16b) , (17c) giving

$$(\overline{\alpha}\mathcal{J}_z + e^{-i\overline{\theta}}\mathcal{J}_+ + e^{i\overline{\theta}}\mathcal{J}_-)^{2k} = (\overline{\alpha}^2\mathcal{J}_0^2 + 2\mathcal{J}_0)^k$$

$$(\overline{\alpha}\mathcal{J}_z + e^{-i\overline{\theta}}\mathcal{J}_+ + e^{i\overline{\theta}}\mathcal{J}_-)^{2k+1} = (\overline{\alpha}^2\mathcal{J}_0^2 + 2\mathcal{J}_0)^k\ (\overline{\alpha}\mathcal{J}_z + e^{-i\overline{\theta}}J_+ + e^{i\overline{\theta}}\mathcal{J}_-) \quad ; \quad k = 0,1,2,... \qquad (22e)$$

we apply the respective qubit state algebraic operations generated by $\mathcal{J}_\pm$, $\mathcal{J}_0$ in equations (20b) , (20c) , (21b) , (21c) and introduce standard trigonometric function expansions to obtain the time evolving spin-up and spin-down antiJC-spinor state vectors in equation (22d) in the explicit form

$$|\overline{\Psi}_{+n}(t)\rangle = e^{-i(-\delta j_{-n})t}(\ (\cos(\ \overline{\mathcal{R}}_{+n}t) - i\ \overline{c}_{+n}\sin(\ \overline{\mathcal{R}}_{+n}t)\ )|\psi_{+n}\rangle - i\ \overline{s}_{+n}e^{i\overline{\theta}}\sin(\ \overline{\mathcal{R}}_{+n}t)|\psi_{-n-1}\rangle)$$

$$|\overline{\Psi}_{-n}(t)\rangle = e^{-i(-\delta j_{+n})t}(\ (\cos(\ \overline{\mathcal{R}}_{-n}t) + i\ \overline{c}_{+n}\sin(\ \overline{\mathcal{R}}_{-n}t)\ )|\psi_{-n}\rangle - i\ \overline{s}_{+n}e^{-i\overline{\theta}}\sin(\ \overline{\mathcal{R}}_{-n}t)|\psi_{+n+1}\rangle) \qquad (22f)$$

where we have introduced the respective Rabi oscillation frequencies $\overline{\mathcal{R}}_{\pm n}$ and interaction parameters $\overline{c}_{\pm n}$, $\overline{s}_{\pm n}$ obtained as

$$\overline{\mathcal{R}}_{\pm n} = 2g\sqrt{\overline{\alpha}^2 j_{\mp n}^2 + 2j_{\mp n}} \qquad ; \qquad \overline{c}_{\pm n} = \frac{2g\overline{\alpha}j_{\mp n}}{\overline{\mathcal{R}}_{\pm n}} \quad ; \quad \overline{s}_{\pm n} = \frac{2g\sqrt{2j_{\mp n}}}{\overline{\mathcal{R}}_{\pm n}} \quad ; \quad \overline{\alpha} = \frac{\overline{\delta}}{2g} \qquad (22g)$$

where the frequency detuning $\overline{\delta} = \omega_0 + \omega$ has been defined in equation (6b). We easily determine orthonormalization relations of the general time evolving antiJC-spinor state vectors in the form

$$\langle \overline{\Psi}_{+n}(t) | \overline{\Psi}_{+n}(t) \rangle = 1 \quad ; \quad \langle \overline{\Psi}_{+n}(t) | \overline{\Psi}_{-n}(t) \rangle = 0 \quad ; \quad \langle \overline{\Psi}_{-n}(t) | \overline{\Psi}_{+n}(t) \rangle = 0 \quad ; \quad \langle \overline{\Psi}_{-n}(t) | \overline{\Psi}_{-n} \rangle = 1$$

$$|u_+|^2 + |u_-|^2 = 1 \qquad ; \qquad \langle \overline{\Psi}(t) | \overline{\Psi}(t) \rangle = 1 \tag{22h}$$

Substituting $| \overline{\Psi}_{\pm n}(t) \rangle$ from equation (22f) into equation (22c) provides the general time evolving state vector $| \overline{\Psi}(t) \rangle$ of the antiJC-spinor. We can now use the time evolving state vector $| \overline{\Psi}(t) \rangle$ to determine the dynamical and statistical properties of the antiJC-spinor, noting that setting $u_- = 0$ or $u_+ = 0$ in equation (22c) gives separate descriptions of the spin-up or spin-down antiJC-spinor provided by the respective time evolving state vectors $| \overline{\Psi}_{+n}(t) \rangle$, $| \overline{\Psi}_{-n}(t) \rangle$.

3.3.1 AntiJC-spinor state eigenvectors, energy eigenvalues and frequency-shifts

We determine the state eigenvectors and energy eigenvalues which characterize the dynamical structure of the antiJC-spinor by using the trigonometric relations $2\cos(\overline{\mathcal{R}}_{\pm n}t) = e^{i\,\overline{\mathcal{R}}_{\pm n}t} + e^{-i\,\overline{\mathcal{R}}_{\pm n}t}$, $2i\sin(\overline{\mathcal{R}}_{\pm n}t) = e^{i\,\overline{\mathcal{R}}_{\pm n}t} - e^{-i\,\overline{\mathcal{R}}_{\pm n}t}$, to reorganize the time evolving spin-up and spin-down state vectors $| \overline{\Psi}_{\pm n}(t) \rangle$ in equation (22f) in the equivalent form

$$| \overline{\Psi}_{\pm n}(t) \rangle = e^{-\frac{i}{\hbar}\overline{\mathcal{E}}^{\,+}_{\pm n}t} | \overline{\Psi}^{\,+}_{\pm n} \rangle + e^{-\frac{i}{\hbar}\overline{\mathcal{E}}^{\,-}_{\pm n}t} | \overline{\Psi}^{\,-}_{\pm n} \rangle \tag{23a}$$

where we have introduced the respective spin-up and spin-down antiJC-spinor state eigenvectors and energy eigenvalues $(| \overline{\Psi}^{\,+}_{\pm n} \rangle \ , \ \overline{\mathcal{E}}^{\,+}_{\pm n})$, $(| \overline{\Psi}^{\,-}_{\pm n} \rangle \ , \ \overline{\mathcal{E}}^{\,-}_{\pm n})$ obtained in the form

$$| \overline{\Psi}^{\,+}_{\pm n} \rangle = \frac{1}{2}\big(\ (1 \pm \overline{c}_{\pm n}) |\psi_{\pm n}\rangle + \overline{s}_{\pm n} e^{\pm i\,\overline{\theta}} |\psi_{\mp n \mp 1}\rangle \ \big) \qquad ; \qquad \overline{\mathcal{E}}^{\,+}_{\pm n} = \hbar(-\delta j_{\mp n} + \overline{\mathcal{R}}_{\pm n})$$

$$| \overline{\Psi}^{\,-}_{\pm n} \rangle = \frac{1}{2}\big(\ (1 \mp \overline{c}_{\pm n}) |\psi_{\pm n}\rangle - \overline{s}_{\pm n} e^{\pm i\,\overline{\theta}} |\psi_{\mp n \mp 1}\rangle \ \big) \qquad ; \qquad \overline{\mathcal{E}}^{\,-}_{\pm n} = \hbar(-\delta j_{\mp n} - \overline{\mathcal{R}}_{\pm n}) \tag{23b}$$

These are eigenvectors of the antiJC-spinor Hamiltonian $\overline{\mathcal{H}}$ satisfying eigenvalue equations

$$\overline{\mathcal{H}} | \overline{\Psi}^{\,+}_{\pm n} \rangle = \overline{\mathcal{E}}^{\,+}_{\pm n} | \overline{\Psi}^{+}_{\pm n} \rangle \qquad ; \qquad \overline{\mathcal{H}} | \overline{\Psi}^{\,-}_{\pm n} \rangle = \overline{\mathcal{E}}^{\,-}_{\pm n} | \overline{\Psi}^{\,-}_{\pm n} \rangle \tag{23c}$$

In equation (23b), we express the Rabi frequency $\overline{\mathcal{R}}_{\pm n}$ defined in equation (22g) in the form

$$\overline{\mathcal{R}}_{\pm n} = 2g\overline{\alpha} j_{\mp n} \overline{\lambda}_{\pm n} \qquad ; \qquad \overline{\lambda}_{\pm n} = \sqrt{1 + \frac{2}{\overline{\alpha}^2 j_{\mp n}}} \tag{23d}$$

and substitute $\delta = \omega_0 - \omega$, $2g\overline{\alpha} = \omega_0 + \omega$ to reorganize the energy eigenvalues in the final form

$$\overline{\mathcal{E}}^{+}_{\pm n} = \hbar j_{\mp n}\big(\ \omega(1 + \overline{\lambda}_{\pm n}) - \omega_0(1 - \overline{\lambda}_{\pm n}) \ \big) \qquad ; \qquad \overline{\mathcal{E}}^{-}_{\pm n} = \hbar j_{\mp n}\big(\ \omega(1 - \overline{\lambda}_{\pm n}) - \omega_0(1 + \overline{\lambda}_{\pm n}) \ \big) \tag{23e}$$

which reveals that the dynamical evolution of the antiJC-spinor is characterized by alternate atom and field mode frequency-shifts towards the blue or red sidebands in the upper or lower energy spectrum. The frequency-shift is associated with the atom-field excitation correlation energy generated by the excitation number correlation operator component $-\hbar(\omega_0 + \omega)\hat{a}^\dagger\hat{a}s_-s_+$ of the antiJC-spinor Hamiltonian. Since it depends on the atom-field mode frequency detuning $\overline{\delta} = \omega_0 + \omega$, the atom-field excitation number correlation operator in the antiJC-spinor never vanishes and the associated frequency-shift phenomenon demonstrated in the energy spectrum in equation (23e) is always observable even under resonance conditions $\omega = \omega_0$, in contrast to the JC-spinor case where the frequency-shift vanishes at resonance.

3.3.2 AntiJC-spinor state population inversion and fluctuations

We determine the antiJC-spinor state population inversion $\mathcal{Z}(t)$ and mean square population inversion $\overline{\mathcal{Z}^2}(t)$ as the mean values of the state population inversion operator $\mathcal{J}_z$ and its square $\mathcal{J}_z^2$ in the general time evolving state $|\overline{\Psi}(t)\rangle$ obtained as

$$\mathcal{Z}(t) = \langle \overline{\Psi}(t) | \mathcal{J}_z | \overline{\Psi}(t) \rangle \qquad ; \qquad \overline{\mathcal{Z}^2}(t) = \langle \overline{\Psi}(t) | \mathcal{J}_z^2 | \overline{\Psi}(t) \rangle \tag{24a}$$

Using $|\overline{\Psi}_{\pm n}(t)\rangle$ from equation (22f), we substitute $|\overline{\Psi}(t)\rangle$ from equation (22c) into equation (24a), expand as appropriate, then use the $\mathcal{J}_z$ algebraic relations from equations (20c) , (21c) and apply the standard $|\psi_{\pm n}\rangle$, $|\psi_{\mp n\mp 1}\rangle$ orthonormalization relations to obtain the population inversion and mean square population inversion in the final form

$$\mathcal{Z}(t) = |u_+|^2 \mathcal{Z}_{+n}(t) + |u_-|^2 \mathcal{Z}_{-n}(t) \qquad ; \qquad \overline{\mathcal{Z}^2}(t) = |u_+|^2 \; \overline{\mathcal{Z}^2_{+n}}(t) + |u_-|^2 \; \overline{\mathcal{Z}^2_{-n}}(t) \tag{24b}$$

where we have identified the spin-up and spin-down antiJC-spinor state population inversion $\mathcal{Z}_{\pm n}(t)$ and mean square population inversion $\overline{\mathcal{Z}^2_{\pm n}}(t)$ determined in the respective time evolving states $|\overline{\Psi}_{\pm n}(t)\rangle$ in the form

$$\mathcal{Z}_{\pm n}(t) = \langle\overline{\Psi}_{\pm n}(t)|\mathcal{J}_z|\overline{\Psi}_{\pm n}(t)\rangle \qquad ; \qquad \overline{\mathcal{Z}^2_{\pm n}}(t) = \langle\overline{\Psi}_{\pm n}(t)|\mathcal{J}_z^2|\overline{\Psi}_{\pm n}(t)\rangle \tag{24c}$$

Substituting $|\overline{\Psi}_{\pm n}(t)\rangle$ from equation (22f) into equation (24c), using equations (20c) , (21c) and applying the appropriate orthonormalization relations as explained above, we obtain the spin-up and spin-down antiJC-spinor state population inversion and mean square population inversion in the explicit form

$$\mathcal{Z}_{\pm n}(t) = \pm j_{\mp n}\big(\; \cos^2(\; \overline{\mathcal{R}}_{\pm n}t) + (\overline{c}^2_{\pm n} - \overline{s}^2_{\pm n})\sin^2(\overline{\mathcal{R}}_{\pm n}t) \; \big) \qquad ; \qquad \overline{\mathcal{Z}^2}_{\pm n}(t) = j^2_{\mp n} \tag{24d}$$

We obtain the antiJC-spinor state population inversion fluctuations from the variance according to the definition

$$(\Delta\mathcal{Z}(t))^2 = \overline{\mathcal{Z}^2}(t) - (\mathcal{Z}(t))^2 \tag{24e}$$

which on substituting the explicit forms of $\mathcal{Z}(t)$, $\overline{\mathcal{Z}^2}(t)$ obtained in equation (24b) and reorganizing as appropriate using the relation $|u_+|^2 + |u_-|^2 = 1$ takes the form

$$(\Delta\mathcal{Z}(t))^2 = |u_+|^2(\Delta\mathcal{Z}_{+n}(t))^2 + |u_-|^2(\Delta\mathcal{Z}_{-n}(t))^2 + |u_+|^2|u_-|^2(\mathcal{Z}_{+n}(t) - \mathcal{Z}_{-n}(t) \;)^2 \tag{24f}$$

where the spin-up and spin-down antiJC-spinor state population inversion fluctuations $(\Delta\mathcal{Z}_{\pm n}(t))^2$ are obtained from the respective variances

$$(\Delta\mathcal{Z}_{\pm n}(t))^2 = \overline{\mathcal{Z}^2_{\pm n}}(t) - (\mathcal{Z}_{\pm n}(t))^2 \tag{24g}$$

which we substitute the results from equation (24d) to determine in explicit form

$$(\Delta\mathcal{Z}_{\pm n}(t))^2 = j^2_{\mp n}\left(\; 1 - (\cos^2(\overline{\mathcal{R}}_{\pm n}t) + (\overline{c}^2_{\pm n} - \overline{s}^2_{\pm n})\sin^2(\overline{\mathcal{R}}_{\pm n}t))^2 \; \right) \tag{24h}$$

Substituting the explicit results from equations (24d) , (24h) into equations (24b) , (24f) as appropriate provides the antiJC-spinor population inversion $\mathcal{Z}(t)$ and population inversion fluctuations $(\Delta\mathcal{Z}(t))^2$ in the general time evolving state $|\overline{\Psi}(t)\rangle$ in explicit form.

We defer the time evolution plots of the antiJC-spinor population inversion and population inversion fluctuations briefly to consider them together with the plots of coherence and coherence fluctuations determined in the next subsection as we relate them to the corresponding plots of the JC-spinor dynamical evolution below.

3.3.3 The antiJC-spinor coherence and coherence fluctuations

The coherence $\mathcal{X}(t)$, $\mathcal{Y}(t)$ and mean square coherence $\overline{\mathcal{X}^2}(t)$, $\overline{\mathcal{Y}^2}(t)$ of the antiJC-spinor in the general time evolving state $|\overline{\Psi}(t)\rangle$ are determined as the mean values and mean square values of the respective coherence dynamical operators $\mathcal{J}_x$, $\mathcal{J}_y$ in equation (15f) according to

$$\mathcal{X}(t) = \langle\overline{\Psi}(t)|\mathcal{J}_x|\overline{\Psi}(t)\rangle \qquad ; \qquad \overline{\mathcal{X}^2}(t) = \langle\overline{\Psi}(t)|\mathcal{J}_x^2|\overline{\Psi}(t)\rangle$$

$$\mathcal{Y}(t) = \langle\overline{\Psi}(t)|\mathcal{J}_y|\overline{\Psi}(t)\rangle \qquad ; \qquad \overline{\mathcal{Y}^2}(t) = \langle\overline{\Psi}(t)|\mathcal{J}_y^2|\overline{\Psi}(t)\rangle \tag{25a}$$

Using $|\overline{\Psi}_{\pm n}(t)\rangle$ from equation (22f), we substitute $|\overline{\Psi}(t)\rangle$ from equation (22c) into equation (25a), expand as appropriate and use the $\mathcal{J}_\pm$ algebraic relations from equations (20b) , (21b) giving

$$\mathcal{J}_x|\psi_{\pm n}\rangle = \sqrt{\frac{j_{\mp n}}{2}} \; |\psi_{\mp n\mp 1}\rangle \quad ; \quad \mathcal{J}_x|\psi_{\mp n\mp 1}\rangle = \sqrt{\frac{j_{\mp n}}{2}} \; |\psi_{\pm n}\rangle$$

$$\mathcal{J}_y|\psi_{\pm n}\rangle = \pm i\sqrt{\frac{j_{\mp n}}{2}} \; |\psi_{\mp n\mp 1}\rangle \quad ; \quad \mathcal{J}_y|\psi_{\mp n\mp 1}\rangle = \mp i\sqrt{\frac{j_{\mp n}}{2}} \; |\psi_{\pm n}\rangle \tag{25b}$$

we apply the $|\psi_{\pm}\rangle$, $|\psi_{\mp n\pm1}\rangle$ orthonormalization relations as appropriate to obtain the coherence and mean square coherence in equation (25a) in the final form

$$\mathcal{X}(t) = |u_+|^2\mathcal{X}_{+n}(t) + |u_-|^2\mathcal{X}_{-n}(t) \quad ; \quad \overline{\mathcal{X}^2}(t) = |u_+|^2 \ \overline{\mathcal{X}^2_{+n}}(t) + |u_-|^2 \ \overline{\mathcal{X}^2_{-n}}(t)$$

$$\mathcal{Y}(t) = |u_+|^2\mathcal{Y}_{+n}(t) + |u_-|^2\mathcal{Y}_{-n}(t) \quad ; \quad \overline{\mathcal{Y}^2}(t) = |u_+|^2 \ \overline{\mathcal{Y}^2_{+n}}(t) + |u_-|^2 \ \overline{\mathcal{Y}^2_{-n}}(t) \qquad (25c)$$

where we have identified the spin-up and spin-down antiJC-spinor coherence $\mathcal{X}_{\pm n}(t)$, $\mathcal{Y}_{\pm n}(t)$ and mean square coherence $\overline{\mathcal{X}^2_{\pm n}}(t)$, $\overline{\mathcal{Y}^2_{\pm n}}(t)$ determined in the respective time evolving states $|\overline{\Psi}_{\pm n}(t)\rangle$ in the form

$$\mathcal{X}_{\pm n}(t) = \langle\overline{\Psi}_{\pm n}(t)|\mathcal{J}_x|\overline{\Psi}_{\pm n}(t)\rangle \quad ; \quad \overline{\mathcal{X}^2_{\pm n}}(t) = \langle\overline{\Psi}_{\pm n}(t)|\mathcal{J}_x^2|\overline{\Psi}_{\pm n}(t)\rangle$$

$$\mathcal{Y}_{\pm n}(t) = \langle\overline{\Psi}_{\pm n}(t)|\mathcal{J}_y|\overline{\Psi}_{\pm n}(t)\rangle \quad ; \quad \overline{\mathcal{Y}^2_{\pm n}}(t) = \langle\overline{\Psi}_{\pm n}(t)|\mathcal{J}_y^2|\overline{\Psi}_{\pm n}(t)\rangle \qquad (25d)$$

The coherence fluctuations in the state $|\overline{\Psi}(t)\rangle$ are obtained as variances according to

$$(\Delta\mathcal{X}(t))^2 = \overline{\mathcal{X}^2}(t) - (\mathcal{X}(t))^2 \quad ; \quad (\Delta\mathcal{Y}(t))^2 = \overline{\mathcal{Y}^2}(t) - (\mathcal{Y}(t))^2 \qquad (25e)$$

which on substituting the results from equation (25c) and reorganizing as appropriate using the relation $|u_+|^2 + |c_-|^2 = 1$ take the form

$$(\Delta\mathcal{X}(t))^2 = |u_+|^2(\Delta\mathcal{X}_{+n}(t))^2 + |u_-|^2(\Delta\mathcal{X}_{-n}(t))^2 + |u_+|^2|u_-|^2(\mathcal{X}_{+n}(t) - \mathcal{X}_{-n}(t))^2$$

$$(\Delta\mathcal{Y}(t))^2 = |u_+|^2(\Delta\mathcal{Y}_{+n}(t))^2 + |u_-|^2(\Delta\mathcal{Y}_{-n}(t))^2 + |u_+|^2|u_-|^2(\mathcal{Y}_{+n}(t) - \mathcal{Y}_{-n}(t))^2 \qquad (25f)$$

where $(\Delta\mathcal{X}_{\pm n}(t))^2$, $(\Delta\mathcal{Y}_{\pm n}(t))^2$, are the spin-up and spin-down antiJC-spinor coherence fluctuations obtained as the variances

$$(\Delta\mathcal{X}_{\pm n}(t))^2 = \overline{\mathcal{X}^2_{\pm n}}(t) - (\mathcal{X}_{\pm n}(t))^2 \quad ; \quad (\Delta\mathcal{Y}_{\pm n}(t))^2 = \overline{\mathcal{Y}^2_{\pm n}}(t) - (\mathcal{Y}_{\pm n}(t))^2 \qquad (25g)$$

We now determine the explicit forms of the spin-up and spin-down antiJC-spinor coherence and coherence fluctuations. Substituting $|\overline{\Psi}_{\pm n}(t)\rangle$ from equation (22f) into equation (25d), using equation (25b) and applying the appropriate orthonormalization relations, we obtain the spin-up and spin-down antiJC-spinor coherence and mean square coherence in explicit form

$$\mathcal{X}_{\pm n}(t) = \pm\sqrt{\frac{j_{\mp n}}{2}} \ \overline{s}_{\pm n}\left(\sin\overline{\theta}\sin(2\overline{\mathcal{R}}_{\pm n}t) + \overline{c}_{\pm n}\cos\overline{\theta}(1 - \cos(2\overline{\mathcal{R}}_{\pm n}t))\right) \quad ; \quad \overline{\mathcal{X}^2_{\pm n}}(t) = \frac{j_{\mp n}}{2}$$

$$\mathcal{Y}_{\pm n}(t) = \mp\sqrt{\frac{j_{\mp n}}{2}} \ \overline{s}_{\pm n}\left(\cos\overline{\theta}\sin(2\overline{\mathcal{R}}_{\pm n}t) - \overline{c}_{\pm n}\sin\overline{\theta}(1 - \cos(2\overline{\mathcal{R}}_{\pm n}t))\right) \quad ; \quad \overline{\mathcal{Y}^2_{\pm n}}(t) = \frac{j_{\mp n}}{2} \qquad (25h)$$

which we substitute into equation (25g) to obtain the explicit form

$$(\Delta\mathcal{X}_{\pm n}(t))^2 = \frac{j_{\mp n}}{2}\left(1 - \overline{s}^2_{\pm n}(\sin\overline{\theta}\sin(2\overline{\mathcal{R}}_{\pm n}t) + \overline{c}_{\pm n}\cos\overline{\theta}(1 - \cos(2\overline{\mathcal{R}}_{\pm n}t)))^2\right)$$

$$(\Delta\mathcal{Y}_{\pm n}(t))^2 = \frac{j_{\mp n}}{2}\left(1 - \overline{s}^2_{\pm n}(\cos\overline{\theta}\sin(2\overline{\mathcal{R}}_{\pm n}t) - \overline{c}_{\pm n}\sin\overline{\theta}(1 - \cos(2\overline{\mathcal{R}}_{\pm n}t)))^2\right) \qquad (25i)$$

Substituting the explicit results from equations (25h) , (25i) into equations (25c) , (26f) as appropriate provides the antiJC-spinor coherence $\mathcal{X}(t)$, $\mathcal{Y}(t)$ and coherence fluctuations $(\Delta\mathcal{X}(t))^2$, $(\Delta\mathcal{Y}(t))^2$ in the general time evolving state $|\overline{\Psi}(t)\rangle$ in explicit form.

Comparing the spin-up antiJC-spinor state algebraic operations in equations (20a)-(20c) with the corresponding spin-up JC-spinor state algebraic operations in equations (8a)-(8c) and similarly, comparing the state algebraic operations for the spin-down antiJC-spinor in equations (21a)-(21c) and the corresponding spin-down JC-spinor in equations (9a)-(9c), reveals an important physical property that the dynamical evolution of the antiJC-spinor and the JC-spinor are similar, but occur in reverse order. The similarity of dynamical evolution is evident in the explicit forms of the respective population inversion and coherence in equations ((12d) , (14h)) and ((24d) , (25h)), but the reverse order dynamical evolution is easily understood according to the physical property already explained earlier that the JC-spinor is formed in the coupling of the atom to the clockwise-rotating component of the field mode in a Jaynes-Cummings interaction mechanism, while the antiJC-spinor is formed in the coupling of the atom to the anticlockwise-rotating component of the field mode in an antiJaynes-Cummings interaction mechanism. We demonstrate the reverse order dynamical evolution by plots of the respective population inversion and coherence against scaled time $\tau = gt$ for initial field mode photon number $n = 1$ as specified together with the parameter values $\overline{\alpha} = \alpha + \frac{\omega}{g}$, $\alpha = \frac{1}{1.31}$, $\frac{\omega}{g} = 0.16$, $\overline{\theta} = \theta = 0$ in Fig.15 , Fig.16 below, noting that we set $u_+ = u_- = \frac{1}{\sqrt{2}}$ in determining the explicit forms for the plots.

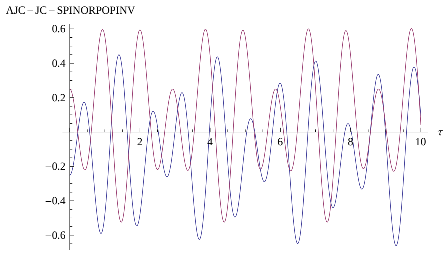

Figure 15: AntiJC-JC-spinor population inversion plots $\mathcal{Z}(\tau) - blue$; $Z(\tau) - red$, $\tau = gt$: $\overline{\alpha} = \alpha + \frac{\omega}{g}$, $\alpha = \frac{1}{1.31}$, $\frac{\omega}{g} = 0.16$; $\overline{\theta} = \theta = 0$; $n = 1$

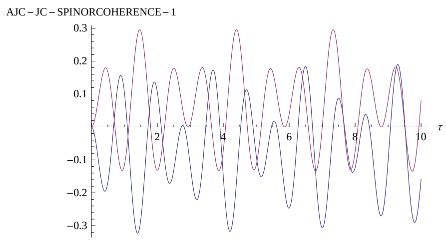

Figure 16: AntiJC-JC-spinor coherence plots $\mathcal{X}(\tau) - blue$; $X(\tau) - red$, $\tau = gt$: $\overline{\alpha} = \alpha + \frac{\omega}{g}$, $\alpha = \frac{1}{1.31}$, $\frac{\omega}{g} = 0.16$; $\overline{\theta} = \theta = 0$; $n = 1$

In Fig.15, we compare the antiJC-spinor population inversion $\mathcal{Z}(\tau)$ from equations $(24b)$, $(24d)$ (blue plot) with the corresponding JC-spinor population inversion $Z(\tau)$ from equations $(12b)$, $(12d)$ (red plot), while in Fig.16, we compare the antiJC-spinor coherence $\mathcal{X}(\tau)$ from equations $(25c)$, $(25h)$ (blue plot) with the corresponding JC-spinor population inversion $X(\tau)$ from equations $(14c)$, $(14h)$ (red plot). It is clear that the blue (antiJC-spinor) and red (JC-spinor) plots evolve in reverse order, but corresponding points on the evolution profiles are slightly displaced from each other due to the fact that, in addition to the flipped quantum numbers $j_{\pm n} \rightarrow j_{\mp n}$, the JC-spinor Rabi oscillation frequency $\mathcal{R}_{\pm n}$ defined in equation $(10h)$ is characterized by the red-sideband (difference) frequency detuning $\delta = \omega_0 - \omega$, while the antiJC-spinor Rabi oscillation frequency $\overline{\mathcal{R}}_{\pm n}$ defined in equation $(22g)$ is characterized by the blue-sideband (sum) frequency detuning $\overline{\delta} = \omega_0 + \omega$ so that the respective Rabi oscillations do not match.

The reverse order dynamical evolution is also evident in the state eigenvector form of the JC-spinor and antiJC-spinor time evolving state vectors $|\Psi_{\pm n}(t)\rangle$, $|\overline{\Psi}_{\pm n}(t)\rangle$ in equations $(11a)$, $(23a)$, respectively, where the field mode and atom frequency-shifts towards the red or blue in the upper or lower energy spectrum in the JC-spinor in equation $(11e)$ occur in reverse order (blue or red) in the corresponding energy spectrum in the antiJC-spinor in equation $(23e)$.

4 Conclusion

Let us now recapture and summarize the basic algebraic and dynamical properties of the quasiparticle excitations in light-matter interactions which we have developed here in the wider framework of the quantum Rabi model, composed of Jaynes-Cummings and antiJaynes-Cummings interaction mechanisms. We have established that a quasiparticle excitation formed in a generalized Jaynes-Cummings or antiJaynes-Cummings interaction mechanism is a composite atom-field spinor, best referred to as a *quasiparticle spinor*, characterized by a closed generalized $SU(2)$ Lie algebra and the associated fermion anticommutation relations. To be specific, we have identified the quasiparticle spinor formed in the generalized Jaynes-Cummings or antiJaynes-Cummings interaction mechanism as a JC-spinor or an antiJC-spinor, respectively. The quasiparticle spinor state annihilation and creation operators ($J_- = \hat{a}^\dagger s_-$, $J_+ = s_+\hat{a}$) or ($\mathcal{J}_- = \hat{a}s_-$, $\mathcal{J}_+ = s_+\hat{a}^\dagger$) are obtained as normal or antinormal order products of the atomic spin and field mode state annihilation and creation operators (s_- , s_+) , ($\hat{a}$, $\hat{a}^\dagger$) in standard notation. Algebraic relations, namely, sum , difference, anticommutation brackets and commutation brackets, of the quasiparticle spinor annihilation and creation operators provide the respective coherence, identity, population inversion and Casimir operators which generate the closed generalized $SU(2)$ Lie algebra and satisfy the associated fermion anticommutation relations of the quasiparticle spinor. It follows from the respective JC-spinor and antiJC-spinor state algebraic operations in equations ((8a)-(8c)) , ((9a)-(9c)) and ((20a)-(20c)) , ((21a)-(21c)) that a quasiparticle spinor may be treated as a generalized two-state spin-$\frac{1}{2}$ particle characterized by an infinite spectrum of integer and half-integer quantum numbers $j_{\pm n} = \frac{1}{2}(n + \frac{1}{2} \pm \frac{1}{2})$ and an infinite spectrum of coupled *photon-carrying spin-up and spin-down qubit state vectors* ($|\psi_{\pm n}\rangle$, $|\psi_{\mp n\pm 1}\rangle$) or ($|\psi_{\pm n}\rangle$, $|\psi_{\mp n\mp 1}\rangle$) specified by atomic spin-up and spin-down state signatures $\pm$ or $\mp$ and field mode photon numbers $n = 0, 1, 2, 3, ..., \infty$. The interpretation of the quasiparticle spinor as a generalized two-state spin-$\frac{1}{2}$ particle is best demonstrated by the coupled qubit state eigenvalue equations ((8c) , (9c)) and ((20c) , (21c)) where we notice that the respective eigenvalues $j_{\pm n} = (n + \frac{1}{2} \pm \frac{1}{2})\frac{1}{2}$, $\pm j_{\pm n} = \pm(n + \frac{1}{2} \pm \frac{1}{2})\frac{1}{2}$ of the spinor identity and state population inversion operators (J_0 , J_z) or ($\mathcal{J}_0$, $\mathcal{J}_z$) are multiples of the eigenvalue $\frac{1}{2}$ of the corresponding spin-$\frac{1}{2}$ particle identity and state population inversion operators (s_0 , s_z) generated on the coupled spin qubit states ($|+\rangle$, $|-\rangle$), while the eigenvalue $j_{\pm n}(j_{\pm n} + 1)$ of the spinor Casimir operator $J^2 = J_0(J_0 + 1)$ or $\mathcal{J}^2 = \mathcal{J}_0(\mathcal{J}_0 + 1)$ generalizes the eigenvalue $\frac{1}{2}(\frac{1}{2} + 1)$ of the corresponding spin-$\frac{1}{2}$ particle Casimir operator $s^2 = s_0(s_0 + 1)$ by the multiplicity factor $(n + \frac{1}{2} \pm \frac{1}{2})$ determined by the field mode photon numbers $n = 0, 1, 2, 3, ..., \infty$. The algebraic operations by the spinor annihilation, creation and population inversion operators ($J_\mp$, J_z) or ($\mathcal{J}_\mp$, $\mathcal{J}_z$) in equations ((8b , 8c) , (9b , 9c)) and ((20b , 20c) , (21b , 21c)) specify the quasiparticle spinor qubit states ($|\psi_{\pm n}\rangle$, $|\psi_{\mp n\pm 1}\rangle$) or ($|\psi_{\pm n}\rangle$, $|\psi_{\mp n\mp 1}\rangle$) as upper or lower qubit states as appropriate, which generalizes the corresponding spin-$\frac{1}{2}$ particle qubit states $|+\rangle$, $|-\rangle$ as upper and lower states, respectively. The quasiparticle spinor closed generalized $SU(2)$ Lie algebra and fermion anticommutation relations in equations ((3a-3d) , (4a-4e)) or ((16a-16d) , (17a-17e)) are generalizations of the corresponding spin-$\frac{1}{2}$ particle closed $SU(2)$ Lie algebra and fermion anticommutation relations in equations (5a)-(5d).

The quasiparticle spinor Hamiltonian is obtained as a generalization of the standard Jaynes-Cummings or antiJaynes-Cummings Hamiltonian including an atom-field excitation number correlation operator component, which arises as an intrinsic algebraic property of the spinor. The Hamiltonian generates a general time evolving entangled state vector describing Rabi oscillations between the spinor qubit state vectors. Expressing the general time evolving state vector as a superposition of two entangled state eigenvectors reveals that the internal dynamics of the spinor is characterized by alternate field mode and atom frequency-shifts towards the red or blue spectra in the upper or lower energy level. The frequency-shift phenomenon is attributed to the atom-field excitation correlation energy, which depends on the frequency detuning $\delta = \omega_0 - \omega$ or $\overline{\delta} = \omega_0 + \omega$ and therefore vanishes at resonance ($\omega = \omega_0$; $\delta = 0$) in the JC-spinor, but remains a permanent dynamical feature in the antiJC-spinor, noting the frequency detuning relation $\overline{\delta} = \delta + 2\omega$ which never vanishes at resonance $\omega = \omega_0$, unless we introduce negative frequency resonance $\omega = -\omega_0$. Dynamical evolution of the mean values and fluctuations of the state population inversion and coherence of the quasiparticle spinor evolving from an initial atom-field superposition state where the atom starts in a spin-up and spin-down superposition state or the field mode starts in a coherent state is characterized by quantum collapses and revivals attributed to the interference of oscillations with different Rabi frequencies.

The quasiparticle spinor interpretation which we have developed in this article provides a useful framework for understanding general dynamical properties and practical applications of light-matter interactions in the quantum Rabi model and the more general Dicke model. The closed generalized $SU(2)$ Lie algebra and fermion algebraic properties means that we can apply the standard algebraic methods for atomic spin coherent and squeezed states [14 , 15 , 16] to determine the coherent and squeezed state properties, as well as the related topological properties, of the quasiparticle spinors in the quantum Rabi and Dicke models. Quasiparticle spinors are useful in formulating generalized spin models of many-body interactions to study ferromagnetic, antiferromagnetic and related quantum phase transition properties in coupled arrays of QED cavities, paralleling widely studied Bose-Hubbard models of many-body interactions based on the bosonic polariton interpretation of the atom-field quasiparticle excitations in [4 , 5]. We note that the closed algebraic properties of the quasiparticle spinor interpretation developed in the present article are exact, in contrast to the approximate algebraic properties of the bosonic polariton interpretation developed in [4 , 5]. The generalized algebraic properties of the quasiparticle spinor also provide great potential for practical applications of the quantum Rabi and Dicke models in the design and implementation of quantum information processing, quantum computation and all related quantum technologies; the infinite spectrum of photon-carrying spin-up and spin-down qubit states ($|\psi_{\pm n}\rangle$, $|\psi_{\mp n\pm 1}\rangle$) of the JC-spinor or ($|\psi_{\pm n}\rangle$, $|\psi_{\mp n\mp 1}\rangle$) of the antiJC-spinor for photon numbers $n = 0, 1, 2, 3, ..., \infty$ can be used to encode a large volume of information, while the respective identity, coherence and state population inversion operators (J_0 , J_x , J_y , J_z) or ($\mathcal{J}_0$, $\mathcal{J}_x$, $\mathcal{J}_y$, $\mathcal{J}_z$) which generate closed $SU(2)$ Lie algebra can be used to construct the appropriate quantum gates according to standard definitions.

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