

## Models for Variable Premiums Payable to Benevolent Funds

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### Abstract

The application of multiple life actuarial calculations have been studied by many authors for instance Elizondo [5] studied the construction of multiple decrement models from associated single decrement experiences. He posits that it is convenient to use the survival functions for the projection of future obligations in cash flows. Bowers [2] studied the actuarial calculations which are common in estate and gift taxation. The actuarial calculation is also common in insurance where stipulated payment called the benefit, one party (the insurer) agrees to pay to the other (the policyholder or his designated beneficiary) a defined amount (the claim payment or benefit) upon the occurrence of a specific loss while the insured pays periodic payment called premium. SACCOs and institutions provide benevolent in terms of insurance against some losses, especially death. Unfortunately such organizations determine their premiums arbitrarily, thus one cannot tell whether such products are degenerating or not, this is because in such bodies benevolent funds and the mainstream operation fund are usually confounded. In this paper we develop models for variable premiums for Saccos and Institutions providing benevolent funds, that is premiums is dependent on the number of beneficiaries. We will use models of joint life, last life and multiple decrements to develop this model.

**Keywords:** Contributor, Beneficiary, Actuarial Present Value, Annuity, Joint life, Multiple decrement and Premiums

## Introduction

Insurance is an agreement where, for a stipulated payment called the benefit, one party (the insurer) agrees to pay to the other (the policyholder or his designated beneficiary) a defined amount (the claim payment or benefit) upon the occurrence of a specific loss. This defined claim payment amount can be a fixed amount or can reimburse all or a part of the loss that occurred, the insurer considers the losses expected for the insurance pool and the potential for variation in order to charge premiums that, in total, will be sufficient to cover all of the projected claim payments for the insurance pool,[1].

Applications of multiple decrements models arise when the amount of benefit payment depends on the mode of exit from the group of active insureds [4]and multiple decrements models are extensions of standard mortality models whereby there is simultaneous operation of several causes of decrement [7].The multiple decrement table is analogous to the life table which is used to calculate survival probabilities and exit probabilities, by mode of exit, for integer ages and durations. the table is used to calculate all survival and exit probabilities for ages within the range of the table[6].The multiple decrement life table is used widely in human actuarial studies to address questions concerning the frequency of occurrence for causes of death and how life expectancy might change if certain causes were eliminated.The conventional life table shows the probability of survivorship of an individual subject to the one undifferentiated hazard of death,[3].

We determine the models for premium which are dependent on the number of survivors of the beneficiaries to be paid by a policyholder to the benevolent fund.We look at different cases involving the contributor and various number of beneficiaries, this will enable us to develop generalized case of contributor and  $n$  beneficiaries. we adopt the model which assumes that each beneficiary pays premium for their benefits. That is beneficiary  $(x_i)$  pays premium  $P(x_i)$  for  $b_i$  benefit.

Here we calculate the annuities independently.

### Case one

Consider the contributor  $x_1$  and one beneficiary  $x_2$

Annuity payable for insurance of  $x_1$  is

$$\bar{a}_{x_1:\overline{n}|} = P \int_0^n v^t {}_t p_{x_1} dt$$

Actuarial present value of the benefit of  $x_1$  is

$$\bar{A}_{x_1:\overline{n}|} = b_1 \int_0^n v^t {}_t p_{x_1} \mu_{x_1}(t) dt$$

Annuity payable for insurance of  $x_2$  is

$$\bar{a}_{x_2:\overline{n}|} = P \int_0^n v^t {}_t p_{x_1 x_2} dt_2$$

Actuarial present value of the benefit of  $x_2$  is

$$\bar{A}_{x_2:\overline{n}|} = b_2 \int_0^n v^t {}_t p_{x_2 x_1} \mu_{x_2}(t) dt_2$$

1. Therefore the premium for  $x_1$  is

$$P(x_1) = \frac{b_1 \int_0^n v^t {}_t p_{x_1} \mu_{x_1}(t) dt}{\int_0^n v^t {}_t p_{x_1} dt_1}$$

2. And the premium for  $x_2$  is

$$P(x_2) = \frac{b_2 \int_0^n v^t {}_t p_{x_2} \mu_{x_2}(t) dt_2}{\int_0^n v^t {}_t p_{x_1 x_2} dt_2}$$

And the premium to paid by the contributor ( $x_1$ ) is;

$$P = P(x_1) + P(x_2)$$

## Case two

Here we have two beneficiaries ( $x_2$ ), ( $x_3$ ) and the Contributor ( $x_1$ ).

Annuity payable for insurance of  $x_1$  is;

$$\bar{a}_{x_1:\overline{n}|} = P \int_0^n v^t {}_t p_{x_1} dt_1$$

Actuarial present value of the benefit for  $x_1$  is;

$$\bar{A}_{x_1:\overline{n}|} = b_1 \int_0^n v^{t_1} {}_{t_1}p_{x_1} \mu_{x_1} dt_1$$

Annuity payable for insurance of  $x_2$  is;

$$\bar{a}_{x_2:\overline{n}|} = P \int_0^n v^{t_2} {}_{t_2}p_{x_1x_2} dt_2$$

Actuarial present value of the benefit for  $x_2$  is;

$$\bar{A}_{x_2:\overline{n}|} = b_2 \int_0^n v^{t_2} {}_{t_2}p_{x_2x_1} dt_2$$

Annuity payable for insurance of  $x_3$  is;

$$\bar{a}_{x_3:\overline{n}|} = P \int_0^n v^{t_3} {}_{t_3}p_{x_1x_3} dt_2$$

Actuarial present value of the benefit for  $x_3$  is;

$$\bar{A}_{x_3:\overline{n}|} = b_3 \int_0^n v^{t_3} {}_{t_3}p_{x_3x_1} dt_3$$

1. Premium payable for the contributor  $x_1$

$$P(x_1) = \frac{b_1 \int_0^n v^{t_1} {}_{t_1}p_{x_1} \mu_{x_1} dt}{\int_0^n v^{t_1} {}_{t_1}p_{x_1} dt_1}$$

2. Premium payable for the beneficiary  $x_2$

$$P(x_2) = \frac{b_2 \int_0^n v^{t_2} {}_{t_2}p_{x_2x_1} dt_2}{\int_0^n v^{t_2} {}_{t_2}p_{x_1x_2} dt_2}$$

3. Premium payable for the beneficiary  $x_3$

$$P(x_3) = \frac{b_3 \int_0^n v^{t_3} {}_{t_3}p_{x_3x_1} dt_3}{\int_0^n v^{t_3} {}_{t_3}p_{x_1x_3} dt_2}$$

The premium payable by the policyholder will therefore be;

$$P = P_{x_1} + P_{x_2} + P_{x_3}$$

**Generally;**

Generally the premium for  $(m - 1)$  becomes;

$$P = \sum_{i=1}^n P(x_i)$$

Where,

$$P(x_i) = \frac{b_i \int_0^n v^{t_i} {}_{t_i}p_{x_1 x_i} \mu_{x_i} dt_i}{\int_0^n v^{t_i} {}_{t_i}p_{x_1 x_i} dt_i}$$

For  $i = 2, 3, \dots, m$  For  $i = 1$

$$P(x_1) = \frac{b_1 \int_0^n v^{t_1} {}_{t_1}p_{x_1} \mu_{x_1} dt_1}{\int_0^n v^{t_1} {}_{t_1}p_{x_1} dt_1}$$

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