Two-Factor Factorial Design:Application in analyzing Differential performance between Single-Sex Schools (Boys Only and Girls only Schools) and Mixed Schools in Compulsory Subjects at K.C.S.E...
A case study in Homa Bay County

by

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Abstract

Despite the recent unabated proliferation of mixed schools, no effort has been directed towards finding out whether they are just as good or even better than single - sex schools. This is in spite of the conventional wisdom which has in the past informed conversion of mixed schools into single - sex schools. (I am yet to come across a case in our country where two oppositely gendered single - sex schools have merged to form a mixed school). This state of affairs begs for attention and it is what motivated the researcher to carry out research in this area. The study applied two-factor factorial design in analyzing differential performance in compulsory subjects between mixed schools and single-sex schools. School type represented one factor while the other factor was represented by subjects. The objectives of the study were to determine whether there is significant effect due to; school type, subject and interaction between school type and subject. School type,subject and interaction between school type and subject were from the analysis of variance, found to have significant effects at α= 5%. The significant interaction effect made it necessary to carry out multiple comparisons. Sheff'e's method revealed statistically significant differences in mean performance in mathematics between single-sex schools and mixed schools. The mean performances in English and Kiswahili for single-sex schools were not, at 5% level of significance, different from those of mixed school using the same (Sheffes) method. The two- factor factorial design model $y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$ was found to be ideal in describing the observed data concerning the performance in compulsory subjects in KCSE.

Chapter 1

Introduction

1.1 Background information

Secondary schools in Kenya can broadly be classified (among other categorizations) as boys schools, girls schools (single-sex schools) or mixed schools (co-educational schools). Most of prominent secondary schools including all national schools are single-sex. Mixed schools are not as prominent and most are either Day schools or partly day and partly boarding schools. Most of these mixed schools have emerged recently as a consequence of the government's effort to provide free Day Secondary school and Subsidized Boarding Secondary School learning. Existence of disparities in performance between these types of schools cannot be denied. To appreciate this fact, one needs only to examine the K.C.S.E results for a given year. Such scrutiny will reveal that the list of the top 100 schools is dominated by National schools all of which are single-sex schools (i.e. Boys only Schools or Girls only Schools). The rest of the positions are taken by county (previously provincial) schools. Only a meagre number of mixed schools occasionally find their way into that list of top 100 schools. Mixed schools are mostly the worst performing schools. It is this state of affairs which prompted the researcher to carry out a study in this area to determine whether the disparities in the performance are statistically significant. The researcher confined his study work in Homa Bay County. In this county, there are two National Schools, a handful number of county schools, with the rest being district schools.

The greatest proportion of schools consists of mixed secondary schools.

1.2 Statement of the Problem

Despite the government's endeavour to attain a 100% transition to secondary, the students may not gain much in the long run. This is due to the poor performance at the end of the four year cycle and the ever widening gap in performance between the well established (mostly single-sex schools) and the emerging ones, most of which are mixed schools. Clearly there is need to investigate the magnitude of these disparities in performance. Of the greatest concern are disparities in performance between single-sex schools and mixed schools with the later constituting the bulk of schools in rural areas. Besides being taken by every candidate at K.C.S.E, compulsory subjects (Namely: English,Kiswahili and Mathematics) determine the final grade attained by a candidate since the grade attained in each is taken into account. They are also used in setting criteria for admission to institutions of higher learning and for career placements. It is for these reasons that the study focused on differential performance in compulsory subjects between mixed schools and boys and girls schools.

1.3 Purpose and Objectives of the Study

The Purpose of this study was to analyze differential performance in compulsory subjects between Mixed Secondary Schools, Boys Schools and Girls Schools in Homa Bay County. The study was guided by the following objectives.

(i) To determine whether there is any significant difference in performance by candidates in different types of schools.

(ii) To determine whether there is any significant difference in performance between subjects.

(iii) To determine whether there is any significant interaction effect between school type and subject performance.

(iv) To carry out multiple comparisons

(v) To fit a model for performance in compulsory subjects

1.4 Hypotheses

The following were the hypotheses of the study.

(i) There is no significant difference in performance by candidates in different types of schools.

(ii) There is no significant difference in performance between subjects.

(iii) There is no significant interaction effect between subjects and school type.

These hypotheses are stated more elaborately in chapter three where testing of hypotheses is discussed in detail.

1.5 Assumptions

In this study the following assumptions were made:

(a) The schools selected for study were of similar social economic status.

(b) Students in the schools under study had similar entry behaviour.

relevance to all stakeholders in the education sector. It will help primary school students and their parents choose the best type of school to join in form one. The parties charged with establishment of secondary schools will find the results of this study valuable in that it endeavoured to show whether one type of school is favourable to students performance as compared to the other. These parties can concentrate on establishing the type of school shown to favour students performance. The results of the study can serve a valuable role in guiding government policy. Based on the findings of the study, the government can make a policy statement concerning the type of new schools to be established. It can also assess the possibility of converting existing schools from one type to another. The study will serve as an impetus for further research. Little research has been done in this area in our country. This study will kindle interest and elicit further research in the area.

1.8 Basic Concepts

In this study, some terms have special meanings or a term is used in a restricted sense. Below are some of these terms and the sense in which they are to be understood.

(i) Compulsory subject: Any of the subjects, English, Kiswahili and Mathematics taken in secondary schools

(ii) Boys school- A secondary school whose student population consists of boys only.

(iii) Girls School- A secondary school whose student population consists of girls only.

(iv) Single-sex (Single-gender) school - a boys school or a girls school.

(v) Mixed (Mixed-gender) school-a school whose student population consists of both boys and girls.

(vi) Co-educational institution a mixed school or mixed gender school as defined in 5 above.

(vii) School type: a boys school or a girls school or a mixed school.

(viii) Subject and compulsory will be used interchangeable.

1.9 Overview of the Chapters

Chapter one deals with the introductory part of the research project.It's main subsection include background to the study,statement of the problem,purpose and objectives of the study,hypotheses of the study,limitations and the significance of the study.Chapter two is concerned with literature review.Here, studies related to the research project are cited.Also cited are the major findings of such studies.Chapter three is dedicated to the methodological aspect of the study.Discussed here is the theory behind the two-factor factorial design and how is related to the analysis in the study. The concepts analysis of variance for two-factor factorial design, the fixed effect model,estimation of model parameters , model adequacy checking, testing of the hypotheses and multiple comparison are discussed in detail.Chapter four is concerned with data analysis and model fitting.The various concepts developed in chapter three are applied in analysing the observed numerical data.Hypotheses are tested and conclusion drawn regarding the populations from which samples were drawn. Chapter fives deals with discussions and conclusion.Its also in this chapter recommendations for further studies are made.

Chapter 2

Literature Review

Various studies have been carried out exploring the relative merits of single gender (single-sex) and mixed gender(mixed-sex) or co-educational schooling. Some have yielded results which favour single gender schooling while others favour mixed gender schooling. For some, single-sex education favoured girls with no clear advantages or with outright disadvantages to boys while in some others the results were the exact opposite of this situation. Yet for some studies single-sex schooling was found to be inferior to co-education in terms of academic success and moulding of students behavior. Thus, there is no consensus on which between single gender and mixed gender schooling is the best. This state of affairs is attested to by the available literature some of which is cited below. Note that most of this literature is from outside our country since local literature is very scanty. Wong [14] examined gender and school type effects on achievement on 45000 Hong Kong students. In Hong Kong, ten percent of public schools are single-sex and thus do not simply cater to elite or religiously affiliated families. These schools do however practice streaming based on gender. In high school, girls are streamed into the stereotypically "female areas of arts and social science whereas boys are generally streamed into the male areas of mathematics and science". A student sample was selected from a list of 1997 graduating exam registers. Wong [14] used a multilevel model of analyzing, which controlled for prior achievement, gender, arts and science stream, co-educational and single-sex schools and the two and three way interaction terms. After controlling for prior achievement, the Authors found that single-sex education benefited girls in English ,the sciences and the arts , while boys from single-sex schools benefited more than their co-educated peers in all subject areas. Wong [14] argue that these are similar findings to those in the UK and Australia. Young and Frazer [15] used secondary data analysis to examine whether there were differences in the science achievement of grade 9 students attending independent, catholic and government , single-sex and co -educational schools in Australia. They found no significant differences in boys or girls overall science achievement in government, catholic and independent co-educational schools, although there were some significant sex differences among individual test questions with girls scoring higher on some items and boys higher on others. Because Social Economic Status (SES) and science achievement were related, the study controlled for school social economic status using a 44 variable indicator of SES derived from data from the Australia Bureau of statistics. Once SES was controlled, girls in single-sex schools demonstrated significantly higher science achievement than their co-educational peers (P<O.10) as did boys in single-sex schools (P<O.05). The author cautioned that higher scores for single-sex private and independent schools and the presence of a significant difference were likely influenced by the absence of government single-sex schools. Baker in 1995 [1] investigated the relationship between grade 12 mathematics achievement and the proportion of single-sex schools in four countries using data from the International Educational Assessments (IEA) second international study(SIMS) hypothesizing that "achievement differences will be largest in countries where the proportion of single-sex schooling is small" using achievement data from two countries: Belgium and New Zealand, which had relatively high percentages of single-sex schools, 68 and 43 respectively and two countries which had relatively low availability of single-sex schools; Thailand with 19 percent and Japan with 14 percent. Baker[1] determined that systems with more even mixes of sex groupings among schools show little or no between- sector achievement difference in contrast to systems with uneven mixes. The authors noted that the higher achievement of girls educated in single-sex schools in Thailand may be due to the fact that in Thailand, most single-sex schools are

in Bangkok and tend to be elite schools for girls , whereas co-educational schools are seen to offer more opportunities for boys. This, they argued, may explain findings of higher achievement differences for girls but not for boys. In contrast, while there was a significant difference in achievement between single-sex schools and co-educational schools in Japan, the effect was reversed. Single-sex schools in Japan produced significantly lower achievement scores than co-educational schools,again, particularly for girls. Baker [1] attributed this results to the context of single-sex schools in Japan, which were oriented towards traditional female roles and less towards academic achievement. Lepore and warren in 1997 [6] Conducted a comparative study of single-sex and co-educational catholic schooling to determine whether or not there were academic and psycho-social differences between students educated in the different environments and whether any differences favoured one gender over the other. Using data from National Educational longitudinal Study [NELS] (1998), Lepore and Warren [6]found no significant differences in achievements once social Economic status and prior achievement were controlled. Nor did they find any significant differences in Psycho-social test scores. Marsh and Rowe in 1996 [8] undertook a re-analysis of studies by Rowe (1995) and Rowe, Nix and Tepper (1986) that compared single-sex and co-educational mathematics classes within a co-educational school. This re-analysis provided no support for the claim that single-sex classes promoted higher achievement for either girls or boys. The re-analysis found no significant differences in achievement or confidence for girls attending single-sex and mixed-sex classes. The achievement of boys attending single-sex classes were significantly greater than those by boys attending mixed classes. Robinson and Smithers in 1999 [10] used standardized government test scores to assess any quantifiable differences in school type effects. The authors found that overall, single-sex schools produce students with higher average scores than co-educational schools. However, after schools were matched for Social Economic status, selectivity and academic tradition, there were no significant differences. Manger and Gjested in 1997 [7] took a slightly different approach to evaluating variables which may influence students performance in mathematics. The authors explored the possibility of existence of a relationship between the ratio of boys to girls and achievements in third grade mathematics classes. Forty nine third grade classes were randomly chosen in the Nowegian City of Bergen, which included a total of 440 girls and 484 boys. Data were gathered using two instruments: 100 item maths quiz based on the national curriculum and a 25 minutes non verbal reasoning test. Although mean scores were typically greater for both boys and girls when the classes had a majority of girls, the Authors found no significant relationship, between the proportion of boys and girls in a class and mean achievement score. Smith [11] conducted a 10 year study of two single-sex schools (one female, one male) in Australia, which had switched to co-educational. Smith was interested in examining possible effects on students self concept and academic achievement due to the change in school type. In terms of academic performance, particular attention was given to the subject areas of English and mathematics. Measures of academic achievement were collected using the results of externally moderated achievement tests at the end of all students grade 10 year, from 1982 to 1986.Smith found no effect on academic achievement on grade 10 test scores in English and mathematics, however, he did note that public examination scores tended to decline in grade12 at the former all girls school. Gillibrand [3] studied 58 girls in a study at a co-educational comprehensive school in England which sought to address the 7:1 gender ratio in physics at the school. 47 of the girls chose to enroll in the girls only physics class, created in the school, with the hope that the number of 14 year old girls who wanted to study physics for general certificate of secondary education (G.C.S.E) would increase along with their confidence and achievement levels. All the students completed a physics anxiety scale to determine their levels of confidence in the subject area and all were interviewed individually. These measurements occurred twice once at the beginning of their course and once at the end. The authors used classroom observations and GCSE scores. Standard tests revealed that although increase in confidence was significant for year 1 and 2 single-sex cohort (P>O.005), there was also a rise for the girls in co-educational cohort which led to a statistically non significant difference between the single-sex and co-educational classes overall, despite a disparity between the number of students enrolled in the two conditions (47) in the single-sex classes and 11 in the co-educational classes. The authors found that increased confidence was positively correlated to GCSE scores and that both higher confidence scores and being in the single-sex classroom were strongly associated with students choice to proceed to A-level physics the following year. From the literature cited above it can be contended that there is no clear verdict concerning which between mixed sex and single sex-schools are best suited for students especially in terms of academic achievements. It is hoped that this study will contribute in enriching the body of knowledge from studies already carried out in this area. It will also serve a pioneering role in the local context where literature in the said area is scantily available.

Chapter 3

Research Methodology

3.1 Location of the Study and Sample Selection

This study was carried out in Homa Bay County. The schools in this county were classified as boys schools, girls schools or mixed schools. From each type of school, a single school was purposively selected. The selected schools were those perceived to be similar in terms of Social-Economic status. It would have been unwise to use a sampling strategy which was likely to result in selection of schools which were widely separated in Social Economic status such as a National School and a Mixed Day School which had been recently established. Selection of a single school per school type was informed by enormity of the ensuing data analysis.

Official records of K.C.S.E results of the selected schools were sought from the respective schools administration. This constituted the data to be used in the analysis. For each of the selected school, a random sample of candidates was drawn from a list of candidates who sat a given compulsory subject for all the subjects. The grade each candidate got was converted to the corresponding points using the conversation table below.

Table 3.1: Conversation table from grades to points

Grade	Points
A	12
A-	11
B+	10
B	9
B-	8
C+	7
C	6
C-	5
D+	4
D	3
D-	2
E	1

3.2 Two-Factor Factorial Design

In a two-factor factorial design, only two factors, say factors$A$ and B are involved. Assuming that there are a levels of factor A and b levels of factor B, then ab is the total number of treatment combinations (cells). A treatment combination (cell) is a level of factor A applied in conjunction with a level of factor B (i.e. ab= the total number of cells). If there are n replicates in each cell, then they may be classified by means of a rectangular array where rows represent levels of one of the factors, say factor A, and columns represent the levels of the other factor(B). The total number of observations (replications) in the experiment is given by abn. Denoting the k^{th} observation ($k = 1, 2, ...n$) taken at the i^{th} level of factor $A(i = 1, 2, ...a)$ and j^{th} level (j=1,2,.....b) of factor B by y_{ijk}, the general data layout for two-factor factorial design is as displayed in table 3.2 below. For example y_{123} is the third observation taken at the first level of factor A and at the second level of factor B.

Definition of some useful symbols used in table 3.2 $y_{ij\cdot}$= sum of observations in the $(ij)^{th}$ cell

$y_{i..}$= Sum of the observations for the i^{th} level of factor A

$y_{.j.}$ = Sum of the observations for the j^{th} level of factor B.

$y_{...}$= Sum of all observations (replications)

$\bar{y}_{ij.}$= Mean of the observations in the $(ij)^{th}$ cell

Table 3.2: General data layout for two-factor design

	Factor B					
Factor A	1	2	....	b	$(y_{i...})$Totals	$(y_{i..})$Means
1	$y_{111}, y_{112}, \dots, y_{11n}$ $y_{11.}$ $\bar{y}_{11.}$	$y_{121}, y_{122}, \dots, y_{12n}$ $y_{12.}$ $\bar{y}_{12.}$	...., ...	$y_{1b1}, y_{1b2}, \dots, y_{1bn}$ $y_{1b.}$ $\bar{y}_{1b.}$	$y_{1..}$	$\bar{y}_{1..}$
2	$y_{211}, y_{212}, \dots, y_{21n}$ $y_{21.}$ $\bar{y}_{21.}$	$y_{221}, y_{222}, \dots, y_{22n}$ $y_{22.}$ $\bar{y}_{a..}$	... ., ...,	$y_{2b1}, y_{2b2}, \dots, y_{2bn}$ $y_{2b.}$ $\bar{y}_{2b.}$	$y_{2..}$	$\bar{y}_{2..}$
. .	. .	. .	. .	. .	 .	
a	$y_{a11}, y_{a12}, \dots, y_{a1n}$ $y_{a1.}$ $\bar{y}_{a1.}$	$y_{a21}, y_{a22}, \dots, y_{a2n}$ $y_{a2.}$ $\bar{y}_{a2.}$	...., ..., ...,	$y_{ab1}, y_{ab2}, \dots, y_{abn}$ $y_{ab.}$ $\bar{y}_{ab.}$	$y_{...}$	$\bar{y}_{a..}$
Totals $(y_{.j.})$	$y_{.1.}$	$y_{.2.}$	.	$y_{.b.}$	$y_{...}$	$y_{...}$
Means$(\bar{y}_{.j.})$	$\bar{y}_{.1.}$	$\bar{y}_{.2.}$	.	$\bar{y}_{.b.}$	$\bar{y}_{...}$	

$\bar{y}_{i..}=$ Mean of the observations for the i^{th} level of factor A.

$\bar{y}_{.j.}=$ Mean of the observations for the j^{th} level of factor B

$\bar{y}_{...}=$ mean of all the abn observations

NOTE: The dot (.) subscript notation implies summation over the subscript that it replaces. Furthermore, the observations in the $(ij)^{th}$ cell constitute a random sample size n from a population that is assumed to be normally distributed with mean μ_{ij} and variance σ^2. All the ab populations are assumed to have the same variance σ^2. Also, it is assumed that the population from which n independent identically distributed observations are taken are combinations of factors and that equal number of observations is taken at each factor combination (cell).

3.2.1 Advantages of Factorial Designs

Factorial designs are more efficient than one factor at a time experiments. Furthermore, a factorial design is necessary where interaction may be present to avoid misleading conclusions. Finally, factorial designs allow the effects of a factor to be estimated at several levels of the other factor yielding conclusions which are valid over a range of experimental conditions [9].

3.2.2 Analysis of Data

The type of statistical analysis employed in factorial design is analysis of variance (ANOVA). Usually statistical software packages such as GENSTAT,SAS,MINITAB, EXCEL e.t.c are employed for the analysis of variance. However, for conceptual and theoretical knowledge and understanding of the method of analysis of two-factor factorial design being used, a manual process is required. Mathematically, the various sums of observations are expressed as follows.

$$y_{ij.} = \sum_{k=1}^{n} y_{ijk} \tag{3.1}$$

$$\bar{y}_{ij.} = \frac{y_{ij.}}{n}; i = 1,2,...a; j = 1,2...b \tag{3.2}$$

$$y_{i..} = \sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk} \tag{3.3}$$

$$\bar{y}_{i..} = \frac{y_{i..}}{bn}; \tag{3.4}$$

$$y_{.j.} = \sum_{i=1}^{a}\sum_{k=1}^{n} y_{ijk} \tag{3.5}$$

$$\bar{y}_{.j.} = \frac{y_{.j.}}{an}; j = 1,2...b \tag{3.6}$$

$$\bar{y}_{...} = \sum_{i=1}^{a}\sum_{i=1}^{b}\sum_{k=1}^{n} y_{ijk} \tag{3.7}$$

$$\bar{y}_{...} = \frac{y_{...}}{abn}; \tag{3.8}$$

The observations in a factorial experiment can be described by means of a model. There are several ways of writing the model for a factorial experiment. The means model is given by;

$$y_{ijk} = \mu_{ij} + \varepsilon_{ijk} \tag{3.9}$$

Where; $i = 1, 2, ..., a, j = 1, 2...., b, k = 1, 2, ..., n$. $\varepsilon_{ijk} =$ The residual or random error (that is , measure of deviation of the observed value (y_{ijk}) in the $(ij)^{th}$ cell from the population mean effect μ_{ij} for the $(ij)^{th}$ cell). The population mean effect for the $(ij)^{th}$ cell, μ_{ij} can also be expressed as:-

$$\mu_{ij} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} \tag{3.10}$$

Where $i = 1, 2, ...a$ and $j = 1, 2, ..., b$. Substituting this into the means model of equation 3.9 gives the effects model,

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \tag{3.11}$$

Where

$$\mu = \frac{1}{ab}\sum_{i=1}^{a}\sum_{j=1}^{b}\mu_{ij} \tag{3.12}$$

is the overall population mean effect;

$$\alpha_i = \mu_i - \mu \tag{3.13}$$

is the effect of the i^{th} level of the row factor A.

$$\beta_j = \mu_j - \mu \tag{3.14}$$

is the effect of the j^{th} level of the column factor B;

$$(\alpha\beta)_{ij} = \mu_{ij} - \mu_i - \mu_j + \mu \tag{3.15}$$

is the interaction effect between the i^{th} level of factor A and the j^{th} level of factor B and

$$\varepsilon_{ijk} = y_{ijk} - \mu_{ij} \tag{3.16}$$

is the residual or random error.

$$\mu_i = \frac{1}{b}\sum_{j=1}^{b} \mu_{ij} \tag{3.17}$$

is the mean effect for the i^{th} level of factor A and finally.

$$\mu_j = \frac{1}{a}\sum_{i=1}^{a} \mu_{ij} \tag{3.18}$$

is the mean effect for the j^{th} level of factor B.

This shows that the effects model of equation 3.11 is partitioned into five under consideration of the assumptions made earlier about the population under the discussion of two-factor factorial design. Taking μ in equation 3.11 to the left hand side of the equal sign(=) gives,

$$y_{ijk} - \mu = \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \tag{3.19}$$

Substituting equations 3.13 ,3.14,3.15 and 3.16 into equation3.19, gives.

$$y_{ijk} - \mu = (\mu_i - \mu) + (\mu_j - \mu) + (\mu_{ij} - \mu_i - \mu_j + \mu) + (y_{ijk} - \mu_{ij}) \tag{3.20}$$

replacing each of the theoretical means μ, μ_i, μ_j and μ_{ij} by their respective unbiased estimators $\bar{y}_{...}$,$\bar{y}_{i..}$,$\bar{y}_{.j.}$ and $\bar{y}_{ij.}$ gives

$(y_{ijk} - \bar{y}_{...}) = (\bar{y}_{i..} - \bar{y}_{...}) + (\bar{y}_{.j.} - \bar{y}_{...}) + (\bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...}) + (y_{ijk} - \bar{y}_{ij.})$ Squaring and summing over i, j and k gives the corrected sum of squares identity.

$$\begin{aligned} \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk} - \bar{y}_{...})^2 &= \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}[(\bar{y}_{i..} \\ &- \bar{y}_{...}) + (\bar{y}_{.j.} - \bar{y}_{...}) \\ &+ (\bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...}) + (y_{ijk} - \bar{y}_{ij.})]^2 \\ &= bn\sum_{i=1}^{a}(\bar{y}_{i..} - \bar{y}_{...})^2 + an\sum_{j=1}^{b}(\bar{y}_{.j.} - \bar{y}_{...})^2 \\ &+ n\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...})^2 \\ &+ \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk} - \bar{y}_{ij.})^2 \end{aligned} \tag{3.21}$$

Note that each of the six cross products on the right hand side is equal to zero. Let

$SS_T =$ total sum of squares

$SS_A =$ sum of squares due to rows or factor A

$SS_B =$ sum of squares due to columns or factor B

$SS_{AB} =$ the sum of squares due to the interaction between factors A and B

$SS_E =$ Sum of squares due residual or random error

Equation 3.21 can be rewritten symbolically as

$$SS_T = SS_A + SS_B + SS_{AB} + SS_E \tag{3.22}$$

For ease in manual computation especially using a desk top calculator,the following formulae are useful.

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{...}^2}{abn} \tag{3.23}$$

$$SS_A = \frac{1}{bn}\sum_{i=1}^{a} y_{i..}^2 - \frac{y_{...}^2}{abn} \tag{3.24}$$

$$SS_B = \frac{1}{an}\sum_{i=1}^{b} y_{.j.}^2 - \frac{y_{...}^2}{abn} \tag{3.25}$$

SS_{AB} is obtained in two stages. First the sum of squares between the ab cell totals, which is called the sum of squares due to subtotals is computed

$$SS_{subtotals} = \frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij.}^2 - \frac{y_{...}^2}{abn} \tag{3.26}$$

This sum of squares also contains SS_A and SS_B . The second step is therefore to compute SS_{AB} as,

$$SS_{AB} = SS_{subtotals} - SS_A - SS_B \tag{3.27}$$

SSE may be computed by subtraction as

$$SS_E = SS_T - SS_{AB} - SS_A - SS_B = SS_T - (SS_{subtotals} - SS_A - SS_B) - SS_A - SS_B \tag{3.28}$$

which reduces to

$$SS_E = SS_T - SS_{subtotals} \tag{3.29}$$

3.2.3 Degrees of Freedom and Mean Squares

Degrees of freedom simply depict the number of independent pieces of information available for computing variability . Generally, it is equal to the sample size (n) minus one, that is $n-1$. According to Gordor and Howard [5], for sum of squares, degrees of freedom are the number of independent elements in the sum of squares concerned. The use of degrees of freedom is to make the sum of squares being calculated an unbiased estimator of the population value. For example, assuming

$SS=\sum_{i=1}^{n}(y_i-\bar{y})^2$

has n elements $(y_1-\bar{y})$, $y_2-\bar{y})$, $(y_3-\bar{y})$, ..., $(y_n-\bar{y})$ these elements are not independent because they sum up to zero, that is

$\sum_{i=1}^{n}(y_i-\bar{y})=0$

Hence only $n-1$ of them are independent meaning that the corresponding sum of squares has $n-1$ degrees of freedom. The numbers of degrees of freedom associated with the sums of squares in equation 3.22 in the corresponding order are; $(abn-1)=(a-1)+(b-1)+(a-1)(b-1)+ab(n-1)$. Ideally since the sample size for the data is abn, the number of degrees of freedom for SS_T is $abn-1$. The main effects factors A and B have a and b levels respectively, implying that SS_A and SS_B have $(a-1)$ and $(b-1)$ degrees of freedom respectively. The interactions number of degrees of freedom is simply equal to the product of the number of degrees of freedom of the two main effects factors A and B, that is $(a-1)(b-1)$. Finally, each of the ab cells has $n-1$ degrees of freedom between the n observations (replications) hence the number of degrees of freedom for SS_E is $ab(n-1)$ Dividing each of the sum of squares on the right hand side of the sum of squares identity ,that is equation 3.22, by the corresponding number of degrees of freedom , gives $MS_A=\frac{SS_A}{a-1}$, $MS_B=\frac{SS_B}{b-1}$, $MS_{AB}=\frac{SS_{AB}}{(a-1)(b-1)}$ and $MS_E=\frac{SS_E}{ab(n-1)}$

Where

$MS_A=$ Sample variance for factor A effects (mean square for factor A effects)
$MS_B=$ Sample variance for factor B effects (Mean square for factor B effects)
$MS_{AB}=$ Mean square for interaction effects between factors A and B.
$MS_E=$ Sample variance for the data (mean square for random error effects)

All these are variance estimates and are independent estimates of σ^2 under the condition that there are no effects α_i, β_j and $(\alpha\beta)_{ij}$. Also

$MS_{Tr} = \frac{SS_{Tr}}{ab-1}$ where MS_{Tr} is the mean square for treatment effects.

3.2.4 Models for Factorial Designs

A model is a theoretical explanation of the phenomenon under study and at the outset, it is usually expressed verbally. To use the model for predictive purposes, this verbal description must be translated into one or more mathematical equations. These equations can be used to determine the value of a specific variable in the model based on the knowledge of the values assumed by the other model variables.

Fitting models in a designed experiment could either be by regression models or effects models. Regression models are particularly useful when one or more of the factors in the experiment are quantitative. However, effects models are used in all cases (Montgomery 2001)

Effects models could be fixed effects models, random effects models or mixed effects models. Fixed effects models are derived when the levels of the factors involved are specifically selected by the experimenter because they are of particular interest. With fixed effects models, inferences are made only on these levels used in the experiment and cannot be extended to cover the whole population . In an experiment whereby the levels of the factor(s) involved are randomly selected from a large group by the experimenter, random effects models are obtained. Mixed effects models result in an experiment whereby the levels of some factor(s) involved are randomly selected and the levels of the other factor(s) are specifically selected by the experimenter. With random effects models and mixed effects models, inferences drawn about the levels cover the whole population from which they are randomly selected [13].

In this study, the levels of both factors involved in the study were specifically selected in advance by the experimenter and therefore the fixed effects model applies. Since regression models, random effects models and mixed effects models are not involved anywhere in the study, no further reference to them will be made in the subsequent discussion.

3.2.5 Anova for Fixed Effects Model

The expected mean squares for two - factor factorial design involving fixed effects are:-

$E(MS_A) = E(\frac{SS_A}{a-1}) = \sigma^2 + \frac{bn\sum_{i=1}^{a}\alpha_i^2}{a-1}$

$E(MS_B) = E(\frac{SS_B}{b-1}) = \sigma^2 + \frac{an\sum_{j=1}^{b}\beta_j^2}{b-1}$

$E(MS_{AB}) = E(\frac{SS_{AB}}{(a-1)(b-1)}) = \sigma^2 + \frac{n\sum_{i=1}^{a}\sum_{j=1}^{b}(\alpha\beta)_{ij}^2}{(a-1)(b-1)}$

$E(MS_E) = E(\frac{SS_E}{ab(n-1)}) = \sigma^2$

The first null hypothesis to be tested is the null hypothesis

(H_o) of no interaction, that is:

$H_o : (\alpha\beta)_{ij} = 0, i = 1, 2, ...a, j = 1, 2, ...b$

H_1 : at least one $(\alpha\beta)_{ij} \neq 0$.

If the hypothesis is not rejected then the analysis is continued by testing for the main effects. For factor A:

$H_o : \alpha_1 = \alpha_2 = ... = \alpha_a = 0$

H_1 : at least one $\alpha_i \neq 0$

And for factor B:

$H_o : \beta_1 = \beta_2 = ... = \beta_b = 0$

H_1 :at least one $\beta_j \neq 0$

. If $H_o : (\alpha\beta)_{ij} = 0$ is rejected , then the null hypothesis for equal treatment combination means is tested.

$H_o : \mu_{11} = \mu_{21} = \mu_{31} .... = \mu_{a1}$

H_1: $\mu_{i1} \neq \mu_{i'1}$ for any $i' \neq i$

i). To test $H_o : (\alpha\beta)_{ij} = 0$, that is, the interaction effects are all equal to zero, the F-ratio: $F_{AB} = \frac{MS_{AB}}{MS_E}$ which is the value of a random variable having the fishers F- distribution with $(a-1)(b-1)$ and $ab(n-1)$ degrees of freedom when

$H_o : (\alpha\beta)_{ij} = 0$ is true, is calculated. Then $H_o : (\alpha\beta)_{ij} = 0$ is rejected at α - level of significance when $F_{AB} > F_{\alpha:(a-1)(b-1),ab(n-1)}$ and it is concluded that interaction is present. Note $F_{\alpha:(a-1)(b-1),ab(n-1)}$ is the table value for an F- distribution with $(a-1)(b-1)$ and $ab(n-1)$ degrees of freedom.

ii) To test $H_o : \alpha_1 = \alpha_2 = ... = \alpha_a = 0$, that is the effects of factor A are all equal to zero; we calculate, $F_A = \frac{MS_A}{MS_E}$ which is the value of a random variable having the F-distribution with $(a-1)$ and $ab(n-1)$ degrees of freedom when $H_o : \alpha_1 = \alpha_2 = ... =, \alpha_a = 0$ is true.

$H_o : \alpha_1 = \alpha_2 = ... = \alpha_a = 0$ is rejected at α - level of significance when $F_A > F_{\alpha:(a-1),ab(n-1)}$ and it is concluded some differences exist between the effects of factor A.

iii) Similarly, to test $H_o : \beta_1 = \beta_2 = ... = \beta_b = 0$, that is, that the effects of factor B are all equal to zero, we compute: $F_B = \frac{MS_B}{MS_E}$ which is the value of a random variable having the F-distribution with $b-1$ and $ab(n-1)$ degrees of freedom when

$H_o : \beta_1 = \beta_2 = ...\beta_b = 0$ is true. $H_o : \beta_1 = \beta_2 = ... = \beta_b = 0$ is rejected at α-level of significance when $F_B > F_{\alpha:(b-1),ab(n-1)}$and it is concluded that some differences between the effects of factor$B$ exist.

iv) If the null hypothesis of no interaction ($H_o : (\alpha\beta)_{ij} = 0$) is rejected, then the F-statistic

$$F_{Tr(ab-1),ab(n-1)} = \frac{MS_{Tr}}{MS_E}$$

is used to test the null hypothesis of no differences among treatment combinations. Any of the various methods of multiple comparisons could also be employed. The initial region for the F-ratio will be the upper tail of the F-distribution.

The analysis of variance ($ANOVA$) table for two-factor factorial design with fixed effects is displayed in table 3.3 below. The table is summarized with the columns containing the source of variation, sum of squares, degrees of freedom, mean squares and calculated F(the test statistic); and the rows containing the sources of variation due to: treatment effect, factor A effect (A), factor B effect (B), interaction effect (AB), error effect and the total effect.

Table 3.3: ANOVA Table

Source of variation	SS	df	MS	F-Ratio
Treatment	$SS_{Tr} = \frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij.}^2 - \frac{y_{...}^2}{abn}$	$(ab-1)$	$MS_{Tr} = \frac{MS_{Tr}}{ab-1}$	$F_{Tr} = \frac{MS_{Tr}}{MS_E}$
A	$SS_A = \frac{1}{bn}\sum_{i=1}^{a} y_{i..}^2 - \frac{y_{...}^2}{abn}$	(a-1)	$MS_A = \frac{SS_A}{a-1}$	$F_A = \frac{MS_A}{MS_E}$
B	$SS_B = \frac{1}{an}\sum_{j=1}^{b} y_{.j.}^2 - \frac{y_{...}^2}{abn}$	(b-1)	$MS_B = \frac{SS_B}{b-1}$	$F_B = \frac{MS_B}{MS_E}$
AB	$SS_{AB} = SS_{Tr} - SS_A - SS_B$	(a-1)(b-1)	$MS_{AB} = \frac{SS_{AB}}{(a-1)(b-1)}$	$F_{AB} = \frac{MS_{AB}}{MS_E}$
ERROR	$SS_E = SS_T - SS_{Tr}$	ab(n-1)	$MS_E = \frac{SS_E}{(ab)(n-1)}$	
Total	$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{...}^2}{abn}$	abn-1		

3.2.6 Graphical Analysis of Two-Factor Factorial Design

A two-dimensional plot of cell means (treatment combination) can provide an insight into the presence of interaction between the two factors (A and B) involved in the study . Significance is indicated by the lack of parallelism of lines, hence the rejection of the null hypothesis (H_o) of interaction in favour of the alternative hypothesis (H_1). Existence of parallelism of the lines indicates that there is no interaction between the two factors (A and B), hence H_o is not rejected.

3.2.7 Multiple Comparisons

When the ANOVA indicates that rows or columns means differ, it is usually of interest to make comparisons between the individual row or column means to discover the specific differences. To do this multiple comparison methods are used.

3.2.7.1 Contrasts

Many multiple comparison methods use the idea of a contrast. In general a contrast is a linear combination of parameters of the form;

$\Gamma = \sum_{i=1}^{a} c_i\mu_i$ where the contrast constants $c_1, c_{2....}, c_a$ sum to zero; that is $\sum_{i=1}^{a} c_i = 0$

3.2.8 Scheff'e's Method for Comparing All Contracts

Scheff 'e's method is used to compare any and all possible contrasts between treatment means. In the Scheff 'e's method, the type 1 error is at most α for any of the possible comparisons. Suppose that a set of m contrasts in the treatment means $\Gamma_u = c_{1u}\mu_1 + c_{2u}\mu_2 + ... + c_{au}\mu_a \; ;u = 1, 2...m$ of interest have been identified. The corresponding contrast in the treatment averages $\bar{y}_{i.}$ is

$C_u = c_{1u}\bar{y}_1 + c_{2u}\bar{y}_2 + ... + c_{au}\bar{y}_a, u = 1, 2....m$ and the standard error of this contrast is

$SC_u = \sqrt{MS_E \sum_{i=1}^{a} (\frac{C_{iu}^2}{n_i})}$

Where n_i is the number of observations in the i^{th} treatment. The critical value against which C_u should be compared is $S_{\alpha u} = SC_{u\sqrt{(a-1)F_{\alpha,a-1,N-a.}}}$

To test the hypothesis that the contrast Γ_u differs significantly from zero, refer C_u to the critical value. If $| Cu |> S_{\alpha,u}$, the hypothesis that the contrast Γ_u equals zero is rejected.

3.2.9 Pairwise Comparisons

Usually one is interested in contrasts of the form $\Gamma = \mu_i - \mu_j$ for all $i \neq j$. Although scheff 'e's method could be easily applied to this problem, it is not the most sensitive procedure for such comparisons [9]. There are several methods which have been designed for Pairwise comparisons between all population means. Here, only one of them-Tukey's method is discussed.

3.2.9.1 Tukey's method

Suppose that following an ANOVA, the hypothesis of equal treatment means has been rejected and one wishes to test all pairwise comparisons;

$H_0 : \mu_i = \mu_j$

$H_1 : \mu_i \neq \mu_j$

For all $i \neq j$

Tukey's procedure makes use of the distribution of the studentized range statistic

$q = \frac{\bar{y}_{max} - \bar{y}_{min}}{\sqrt{\frac{MS_E}{n}}}$

Where $\bar{y}_{max}$ and $\bar{y}_{min}$ are the largest and smallest sample means respectively, out of a group of a sample means. For equal sample sizes, Tukey's test declares two means significantly different if the absolute value of their sample differences exceeds, $T_\alpha = q_{\alpha(a,f)}\sqrt{\frac{MS_E}{n}}$

Where α is the level of significance, a is the number of sample means and f is the number of degrees of freedom associated with the MS_E and n is the sample size. When the sample sizes are not equal, the above equation becomes

$T_\alpha = \frac{q_{\alpha,(a,f)}}{\sqrt{(2)}}\sqrt{MS_E}(\frac{1}{n_i} + \frac{1}{n_j})$

The unequal version is sometimes called the Turkey-Kramar procedure.

Note that when interaction is significant, comparisons between the means of one factor (e.g. A) may be obscured by the AB interaction. One approach to this situation is to fix the other factor (B) at a specific level and apply Tukey's test to the means of factor A at that level. It is assumed that the best estimate of the error variance is the MS_E from the ANOVA table, utilizing the assumption that the experimental error variance is the same over all treatment combinations.

3.2.10 Model Building

Model building entails the development of prediction equations (statistical models) by statistical or mathematical methods from experimental data. As earlier noted, the formula for effects model is given by $y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$, where $i = 1, 2..., a, j = 1, 2...b, k = 1, 2...n$, μ is the overall mean effect, α_i and β_j are the fixed treatment effects of factors A and B respectively and are defined as the deviations from the overall mean effect μ, hence $\sum_{i=1}^{a} \alpha_i = 0$ and $\sum_{j=1}^{b} \beta_j = 0$. Also $(\alpha\beta)_{ij}$ is the fixed interaction effect of factors A and B in the $(ij)^{th}$ cell and is defined in such a manner that $\sum_{i=1}^{a}(\alpha\beta)_{ij}= \sum_{j=1}^{b}(\alpha\beta)_{ij} = 0$. ε_{ijk} is the measure of the deviation of the observed value y_{ijk} in the $(ij)^{th}$ cell from μ_{ij}

3.2.11 Estimation of the Model parameters

The estimation of the parameters of the effects model $y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$ is done by using the least squares method . In summary, if there are a levels of factor A and b levels of factor B, then the model has $(1 + a + b + ab)$ parameters to be estimated and there are $(1 + a + b + ab)$ normal equations which are given by:

$$\mu : abn\hat{\mu} + bn\sum_{i=1}^{a} \hat{\alpha}_i + an\sum_{j=1}^{b} \hat{\beta}_j + n\sum_{i=1}^{a}\sum_{j=1}^{b} (\hat{\alpha\beta})_{ij} = y_{...}; where i = 1, 2...a, j = 1, 2, ...b \tag{3.30}$$

$$\alpha_i : bn\hat{\mu} + bn\hat{\alpha}_i + n\sum_{j=1}^{b}\hat{\beta}_j + n\sum_{j=1}^{b}(\hat{\alpha\beta})_{ij} = y_{i..} \tag{3.31}$$

where j= 1, 2,.....b

$$\beta_j : an\hat{\mu} + n\sum_{i=1}^{a}\hat{\alpha}_i + an\hat{\beta}_j + n\sum_{i=1}^{a}(\hat{\alpha\beta})_{ij} = y_{.j.} \tag{3.32}$$

where i=1,2,.... a

$$(\alpha\beta)_{ij} : n\hat{\mu} + n\hat{\alpha}_i + n\hat{\beta}_j + n(\hat{\alpha\beta})_{ij} = y_{ij.} \tag{3.33}$$

Applying the assumptions $\sum_{i=1}^{a}\alpha_i = 0$

$\sum_{j=1}^{b}\beta_j = 0$

$\sum_{i=1}^{a}(\alpha\beta)_{ij} = 0$ and $\sum_{j=1}^{b}(\alpha\beta)_{ij} = 0$

gives

$$\hat{\mu} = \bar{y}_{...} \tag{3.34}$$

$$\hat{\alpha}_i = \bar{y}_{i..} - \bar{y}_{...} \tag{3.35}$$

$$\hat{\beta}_j = \bar{y}_{.j.} - \bar{y}_{...} \tag{3.36}$$

and

$$(\hat{\alpha\beta})_{ij} = \bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...} \tag{3.37}$$

substituting these values in the equation, $\hat{y}_{ijk} = \hat{\mu} + \hat{\alpha}_i + \hat{\beta}_j + (\hat{\alpha\beta})_{ij}$

gives $\hat{y}_{ijk} = \bar{y}_{...} + (\bar{y}_{i..} - \bar{y}_{...}) + (\bar{y}_{.j.} - \bar{y}_{...}) + (\bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...}) = \bar{y}_{ij.}$

This means that, the k^{th} observation in the $(ij)^{th}$ cell is estimated by the average of the n observations (replicates) in that cell.

3.2.12 Model Adequacy Checking

Before the conclusions from the analysis of variance are adopted, the adequacy of the model should be checked. The primary diagnostic tool in model adequacy checking is the residual analysis which is mostly done by graphical analysis in different forms and simply called residual plots. In [13] a residual is defined as essentially an error in the fit of a model . The residuals for a two-factor factorial model are given by

$$\varepsilon_{ijk} = y_{ijk} - \hat{y}_{ijk}$$

where $\hat{y}_{ijk}$ is the estimator of y_{ijk} given by $\hat{y}_{ijk} = \bar{y}_{ij.}$. This implies that

$$\varepsilon_{ijk} = y_{ijk} - \bar{y}_{ij.} \tag{3.38}$$

The residual plots are:

(i) the normal probability plot of the model

(ii) residual plot in time sequence used to check independence assumption on the error and

(iii) plot of the residuals versus fitted values $\hat{y}_{ijk}$), used to check constancy of variance.

For the normal probability plot of residuals, if the underlying error distribution is normal, then the plot exhibits some kind of linearity, hence the adequacy of the model. In the case of residual plots in time sequence, when the points in the graph are uniformly spread out about the mean of the residuals zero, then there is no reason to suspect any violation of the independence assumption, hence the adequacy of the model. As for the plot of residuals verses the fitted values, when the points are uniformly scattered about the mean, zero, and do not portray any obvious pattern, then the variance is constant and the model is adequate. Montgomery [9] determined that, if the model is adequate, the residuals should be structure-less, that is, they should contain no obvious patterns. However, a very common defect that often shows up on the normal probability plots is one residual being much larger than the others, and this can seriously distort the analysis of variance. This residual is called an outlier. Mostly, the cause of the outlier is such human error as calculation error, data coding error, or copying error. However, a suspected outlier could be checked by examining the standardized residuals value (d_{ijk}) given by, $d_{ijk} = \frac{\varepsilon_{ijk}}{\sqrt{MS_E}}$ A residual value (d_{ijk}) bigger than 3 in absolute value is a potential outlier which can cause a serous distortion to the conclusions drawn from the ANOVA.

3.3 Relating The Analysis In The Study To Two-Factor Factorial Design

In this study, the two factors of a two-factor factorial design were taken to be school type and subject, with school type being the row factor (factor A) and subject being the column factor (factor B). There were three levels of factor A ($i.e$; $a = 3, i = 1, 2, 3$) namely; (1) Boys school (2) Girls school and (3) mixed schools, which were for the sake of convenience represented by the numbers 1,2 and 3 respectively. Similarly there were three levels of factor B ($i.e$; $b = 3, j = 1, 2, 3$) namely: (1) English (2) Kiswahili and (3) Mathematics, which were represented by the numbers 1,2 and 3 respectively. The observations in the study were the points (see table 3.1) scored by a candidate as sampled from a list of candidates from a given school who had taken a particular subject. A constant sample of size 25 ($i.e$; $n = 25$) was used for every school type and subject combination. Justification of using a constant sample size of 25 will be addressed shortly. The following formula was used to determine the combined sample size for all $ab(i.e : 3 \times 3 = 9)$ cells.

$$n_c = \frac{Z^2 \times p \times q}{d^2}$$

where n_c=the desired sample size if the population is large [2] i.e. greater than 10,000
z=the standard normal deviate at the required confidence level.
p=the proportion in the target population assumed to have the characteristics being mea-

sured.

$q = 1 - p$

d=the absolute precision. Since there is no estimate available of the proportion assumed to have the characteristic of interest, 50% was used. Taking $p = 50\% = 0.5, \Longrightarrow q = 0.5$

$\alpha = 0.05; z = 1.96$ gives. $n_c = \frac{1.96^2 \times 0.5 \times 0.5}{0.05^2} = 384$

The populations (number of candidates of the sampled schools) were as follows: Boys school =179

Girls school=163

Mixed school=171

Total (N) =513

Since the total population is small,(less 10,000) the following correction formula was applied [2].

$n_T = \frac{n_c}{1+\frac{n_c}{N}} = \frac{384}{1+\frac{384}{513}} = 219.6168$

The sample size (n) per cell is computed from: abn=3 × 3n=219.6168

$\Rightarrow$ n= $\frac{219.6168}{9}$

n= 24.4019

≈ 25

Proportional allocation would have yielded the following values of sample sizes. $n_{11} = n_{12} = n_{13} = \frac{179 \times 219.6168}{513 \times 3} = 24.5479 \approx 26$

$n_{21} = n_{22} = n_{23} = \frac{163 \times 219.6168}{513 \times 3} = 23.260 \approx 24$

$n_{31} = n_{32} = n_{33} = \frac{171 \times 219.6168}{513 \times 3} = 24.402 \approx 25$

These values are not much different from the constant value of 25, hence there was no harm in using this constant value for the sample size. With the above brief description, everything else neatly falls in its rightful place in what was discussed earlier regarding two-factor factorial design.

Chapter 4

Data analysis and Model Fitting

4.1 Data presentation and analysis

Following the procedures described in sections 3.1 and 3.3 data were collected and displayed in two-factor factorial design layout as follows.

Table 4.1: KCSE(2011) Performance (in points) data

Factor A	Factor B				
School type	English	Kiswahili	Mathematics	$y_{i..}$	$\bar{y}_{i..}$
	j=1	j=2	j=3	Totals	Means
Boys school	8,10,9,9,10	10,7,11,10,7	11,12,6,11,12		
i=1	5,8,9,9,9	8,9,12,10,11	6,10,12,10,8		
	11,10,10,7,10	10,9,11,11,12	8,5,10,6,11		
	9,10,10,9,8	7,9,12,10,10	12,6,7,11,1		
	10,12,9,8,9	11,12,10,9,11	9,6,11,8,11		
	$y_{11.} = 228$	$y_{12.} = 249$	$y_{13.} = 227$	704	
	$\bar{y}_{11.}$=9.12	$\bar{y}_{12.}$=9.96	$\bar{y}_{13.} = 9.08$		9.39
Girls school	9,8,6,8,6	7,6,8,6,9	8,4,7,1,3		
i=2	8,9,8,9,8	8,10,12,7,7	2,9,6,3,4		
	10, 9,8,7,7	7, 5,9,8,10	4,2,8,7,4		
	10,7, 8,5,10	10,8,12,10,7	6,7 ,4 ,5,9		
	7, 8,8 ,9,8	9, 5 , 8 ,8,11	2,7, 3, 8, 5		
	$y_{21.} = 200$	$y_{22.} = 207$	$y_{23.} = 128$	535	
	$\bar{y}_{21.}$=8	$\bar{y}_{22.}$=8.28	$\bar{y}_{23.} = 5.12$		7.13
Mixed school	8,8,8 ,8, 10	10,10,7,10,11	7,4, 6 , 3 , 4		
i=3	9,9,9, 8 ,8	10, 9,9,7,8	9,7,2,8,4		
	10, 7,6,7,10	9,10,10, 9, 9	4,6 ,9,5,9		
	8,9,9,10,8	9,6,10,10,8	5, 8,6,1,5		
	9,11,9,8,7	5,9, 9,10,10	6,7,3, 8,7		
	$y_{31.} = 213$	$y_{32.} = 224$	$y_{33.} = 143$	580	
	$\bar{y}_{31.}$=8.54	$\bar{y}_{32.}$=8.96	$\bar{y}_{33.} = 5.72$		7.73
Total ($y_{.j.}$)	641	680	498	1819	23.25
Means($y_{.j.}$)	8.55	9.07	6.64	24.26	8.08

In the above table, school type is the row factor $(factorA)$ while subject is the column factor $(factorB)$. The levels of factor A are boys school $(i = 1)$, girls school $(i = 2)$ and mixed school $(i = 3)$ (that is $a = 3, i = 1, 2, 3$) while the levels of factor B are English $(j = 1)$, Kiswahili $(j = 2)$ and mathematics $(j = 3)$ (that is $b = 3, j = 1, 2, 3$). A visual examination of the data, especially that of the totals and means reveals that the performance for the boys school $(i = 1)$ was the best in all the three subjects followed by that of the mixed school $(i = 3)$ and lastly that of the girls school $(i = 2)$, while subject wise, performance in kiswahili $(j = 2)$ was the best for all the three school types followed by English $(j = 1)$ and lastly that in mathematics $(j = 3)$. Visual examination alone is inadequate in making decisions concerning such data. The relevant statistical analysis, in this case, analysis of variance $(ANOVA)$ is required. In order to realize the analysis of variance table for the two-factor factorial design, it is necessary to compute the various sums of squares. This working is set out as follows;

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{...}^2}{abn}$$
$$= (8^2 + 9^2 + ... + 8^2 + 7^2) - \frac{1819^2}{3\times3\times25}$$
$$= 15927 - 14705.604$$
$$= 1221.396$$

$$SS_A = \frac{1}{bn}\sum_{i=1}^{a} y_{i..}^2 - \frac{y_{...}^2}{abn}$$
$$= \frac{1}{3\times25} \times (704^2 + 535^2 + 580^2) - 14705.604$$
$$= 14950.4667 - 14705.604$$
$$= 204.276$$

$$SS_B = \frac{1}{an}\sum_{i=1}^{b} y_{.j.}^2 - \frac{y_{...}^2}{abn}$$
$$= \frac{1}{3\times25} \times (641^2 + 680^2 + 498^2) - 14705.604$$
$$= 14950.667 - 14705.604$$
$$= 244.862$$

$SS_{AB} = SS_{subtotals} - SS_A - SS_B$

$= \frac{1}{n}\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij.}^2 - \frac{y_{...}^2}{abn} - SS_A - SS_B = \frac{1}{25}[228^2 + 249^2 + ... + 224^2 + 143^2] - \frac{1819}{225} -$ $204.276 - 244.862$

$= 524.036 - 204.276 - 244.862$

74.898

$SS_E = SS_T - SS_{subtotals} = 1221.396 - 524.036$

$= 697.360$

Table 4.2: ANOVA Table of performance in compulsory subjects

Source of variation	SS	df	MS	Calculated F
Treatment	524.036	8	65.505	20.286
School type(factor A)	204.276	2	102.138	31.636
Subject (factor B)	244.862	2	122.431	37.922
school type subject interaction (AB)	74.898	4	18.725	5.800
ERROR	697.360	216	3.229	
Total	1221.396	224		

In carrying out statistical tests of hypotheses, a 5% ($\alpha = 0.05$) level of significance was used throughout in this study.

To test the hypothesis of no interaction between school type and subject, the computed F-ratio, $F_{AB} = 5.800$ from the $ANOVA$ table above was compared with the table value $F_{\alpha:(a-1)(b-1),ab(n-1)} = F_{0.05,4,216} = 2.3719$ from the table for F- distribution in appendix 5.2. Note that it was assumed that $F_{0.05,4,216} = F_{0.05,4,\infty} = 2.3719$ since extrapolation would give a value for $F_{0.05,4,216}$ smaller than $F_{0.05,4,\infty}$ which is not reasonable. Since $F_{AB} = 5.800 > F_{0.05,4,216} = 2.3719$, the null hypothesis of no interaction between school type and subject is rejected. It is therefore concluded that there is statistical evidence that there is interaction between school type (factor A) and subject (factor B). This gives the general indication that performance is dependent on both school type and subject.

The analysis was continued by testing the null hypothesis of no difference among treatment combinations. From the table for F distribution in appendix 5.2, $F_{\alpha,(ab-1),ab(n-1)} = F_{0.05,8,216} = 1.9384$. Since the calculated F for treatment combination means difference (F_{Tr}) from the ANOVA table 4.2 is equal to $20.286 > F_{0.05,8,216} = 1.9384$ the treatment combination variance is significant. Hence the null hypothesis of no difference among treatment means is rejected. Note that it was assumed that $F_{0.05,8,216} = F_{0.05,8,\infty} = 1.9384$ since extrapolation of $F_{0.05,8,120}$ would give a value smaller than $F_{0.05,8,\infty}$ which is not reasonable.

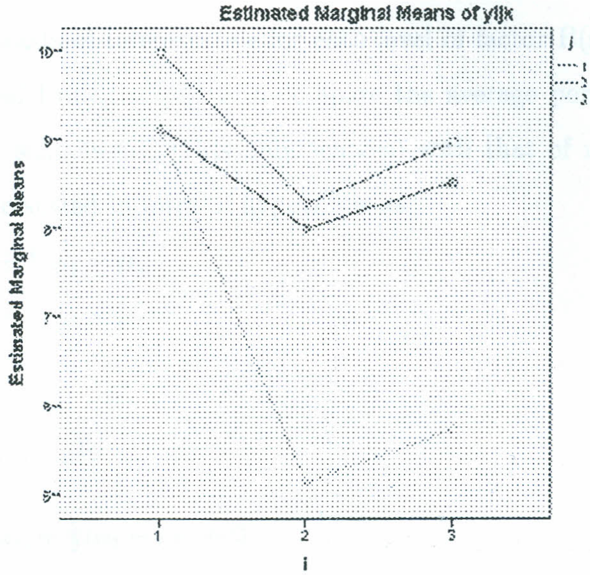
To test the two null hypotheses that effects due to the two main factors A and B are equal to zero, the calculated F - ratios, $F_A = 31.636$ and $F_B = 37.922$ were compared with the respective table values $F_{\alpha,(a-1),ab(n-1)} = F_{0.05,2,216} = 2.9957$ and $F_{\alpha,(b-1),ab(n-1)} = F_{0.05,2,216} = 2.9957$. Since $F_A = 31.636 > F_{0.05:2,216} = 2.9957$ and $F_B = 37.922 > F_{0.05,2,216} = 2.9957$ both null hypotheses were rejected and it was concluded that the effects due to the two main factors namely; school type(factor A) and subject(factor B) are significant.

Since there is significant interaction effect between school type and subject, interpretation of the analysis is not straight forward.

4.1.1 Graphical Analysis of Performance In Compulsory Subjects

To assist in interpreting the results of this study, a graph of the average responses (i.e cell means) at each treatment combination was constructed as shown in figure 4.1.1 below.

Figure 4.1: Estimated marginal means of y_{ijk} plot of performance



The significant interaction is indicated by the lack of parallelism between the lines. In general performance decreases for all the subjects from school type $i = 1$(boys school) to school type $i = 2$(girls school) and then mildly rises from the school type $i = 2$ to school type $i = 3$ (mixed school). The rate of decreases is greatest in mathematics ($j = 3$) followed by Kiswahili ($j = 2$) and lastly English ($j = 1$) as indicated by the steepness of the lines.

Similarly as one moves from school type $i = 2$, to school type $i = 3$, the rate of increase in performance is greatest for mathematics $i = 3$ followed by Kiswahili ($i = 2$) and finally English ($i = 1$). This confirms that performance depends on school type and subject.

4.2 Multiple Comparisons for Performance in Compulsory Subjects

4.2.1 Scheff'e's method

Three contrasts of interest, one for each level of factor B(subject) were identified. They were derived from the desire to compare the average performance of single-sex schools (Boys only schools and Girls only schools) with that of mixed school for each subject. This can be stated in form of hypothesis as,

$H_{0j} : \frac{1}{2}\mu_{1j} + \frac{1}{2}\mu_{2j} = \mu_{3j}$

$H_{1j} : \frac{1}{2}\mu_{1j} + \frac{1}{2}\mu_{2j} \neq \mu_{3j}$

$(j = 1, 2, 3)$

Or equivalently

$H_{oj} : \frac{1}{2}\mu_{1j} + \frac{1}{2}\mu_{2j} - \mu_{3j} = 0$

$H_{1j} : \frac{1}{2}\mu_{1j} + \frac{1}{2}\mu_{2j} - \mu_{3j} \neq 0$

This can be expressed in terms of a contrast as

$H_{oj} : \Gamma_j = 0$

$H_{1j} : \Gamma_j \neq 0$

Where,

$\Gamma_j = \sum_{i=1}^{3} c_i\mu_{ij} = \frac{1}{2}\mu_{1j} + \frac{1}{2}\mu_{2j} - \mu_{3j}$

Note that the contrast coefficients c_i sum to zero i.e

$\sum_{i=1}^{3} c_i = \frac{1}{2} + \frac{1}{2} - 1 = 0,$

satisfying the fundamental requirement for Γ_j to be a contrast.

The corresponding contrast in the treatment average $\bar{y}_{ij.}$ is

$C_j = \sum_{i=1}^{3} c_i\bar{y}_{ij.} = \frac{1}{2}\bar{y}_{1j.} + \frac{1}{2}\bar{y}_{2j.} - \bar{y}_{3j.}$

and the standard error of this contrast is

$S_{Cj} = \sqrt{MS_E \sum_{i=1}^{a}(\frac{c_{ij}^2}{n_i})}$

The critical value against which C_j should be compared is.

$S_{\alpha,j} = S_{Cj}\sqrt{(a-1),(F_{(\alpha,(a-1)),(N-a)})}$

If $|c_j| > S_{\alpha,j}$, the hypothesis that the contrast Γ_j equals zero is rejected. The three identified contrasts Γ_1, Γ_2 and Γ_3 corresponding to the subject levels; English ($j = 1$), Kiswahili ($j = 2$) and Mathematics ($j = 3$) respectively were

$\Gamma_1 = \frac{1}{2}\mu_{11} + \frac{1}{2}\mu_{21} - \mu_{31}$

$\Gamma_2 = \frac{1}{2}\mu_{12} + \frac{1}{2}\mu_{22} - \mu_{32}$

and $\Gamma_3 = \frac{1}{2}\mu_{13} + \frac{1}{2}\mu_{23} - \mu_{33}$

The numerical values of these contrasts are

$C_1 = \frac{1}{2}\bar{y}_{11.} + \frac{1}{2}\bar{y}_{21.} - \bar{y}_{31.} = \frac{1}{2} \times 9.12 + \frac{1}{2} \times 8 - 8.52 = 0.04$

$C_2 = \frac{1}{2}\bar{y}_{12.} + \frac{1}{2}\bar{y}_{22.} - \bar{y}_{32.} = \frac{1}{2} \times 9.96 + \frac{1}{2} \times 8.28 - 8.96 = 0.16$

$C_3 = \frac{1}{2}\bar{y}_{13.} + \frac{1}{2}\bar{y}_{23.} - \bar{y}_{33.} = \frac{1}{2} \times 9.08 + \frac{1}{2} \times 5.12 - 5.72 = 1.38$

$S_{C1} = S_{C2} = S_{C3} = \sqrt{MS_E \sum_{i=1}^{3}(\frac{c_{ij}^2}{n_i})}$

$= \sqrt{3.229(\frac{.25+.25+1}{25})} = 0.4402$

$S_{\alpha_j} = S_{0.05,j} = S_{0.05,1} = S_{0.05,2} = S_{0.05,3}$

$= SC\sqrt{(a-1)(F_{\alpha,(a-1),(N-a)})}$

$= 0.4402[\sqrt{2 \times F_{0.05,2,212}}]$

$= 0.4402 \times 2.9957$

$= 1.0775$

Because $C_1 = 0.04 < S_{0.05,1} = 1.0775$,

it was concluded that the mean performance of single - sex schools in English is not significantly different from that of mixed schools. Similarly, since $C_2 = 0.16 < S_{0.05,2} = 1.0775$, it was concluded that the average performance of single - sex schools in Kiswahili is not significantly different from that of mixed schools.

Since $C_3 = 1.38 > S_{0.05,3} = 1.0775$ it was concluded that $\Gamma_3 = \frac{1}{2}\mu_{13} + \frac{1}{2}\mu_{22} - \mu_{33}$ does not equal zero; that is, it was concluded that the mean performance of single - sex schools in Mathematics is significantly different from the mean performance of mixed schools.

4.2.2 Tukey's Method

Tukey's method was used to carry out pairwise comparisons between the means of factor A (school type). Since interaction was significant, this was done when factor B (subject) was fixed at its respective levels, $j = 1$ (English) $j = 2$ (Kiswahili) and $j = 3$ (Mathematics). The test statistic (T_α) for the turkey's test is given by

$T_\alpha = q_{\alpha,(a,f)}\sqrt{\frac{MS_E}{n}}$;
$T_{0.05} = q_{0.05(3,216)}\sqrt{\frac{3.229}{25}}$
$= 3.3392 \times 0.3594$
$= 1.2001$ Note that $q_{0.05(3,216)} = 3.3392$ was obtained by interpolation in the interval between $q_{0.05(3,120)}$ and $q_{0.05(3,240)}$. When factor B (subject) was fixed at $j = 1$(English), the means for Boys schools ($i = 1$), Girls schools ($i = 2$) and mixed schools(i=3) were: $\bar{y}_{11.} = 9.12$, $\bar{y}_{21.} = 8.00$ and $\bar{y}_{31.} = 8.52$ respectively. When factor B(subject) was fixed at j=2(Kiswahili),the mean performance for Boys schools ($i = 1$), Girls schools ($i = 2$) and mixed schools ($i = 3$) were $\bar{y}_{12.}$ =9.96, $\bar{y}_{22.} = 8.28$ and $\bar{y}_{32.}$ =8.96 respectively. And when factor B was fixed at $j = 3$(Mathematics) the mean performance for boys schools ($i = 1$), girls schools ($i = 2$) and mixed schools ($i = 3$) were $\bar{y}_{13.} = 9.08, \bar{y}_{23.} = 5.12$ and $\bar{y}_{33.} = 5.72$ respectively.

Any pair of mean performances that differ in absolute value by more than $T_{0.05} = 1.2001$ would imply that the corresponding pair of population means are significantly different. For factor B fixed at $j = 1$(English), the absolute differences in mean performance were as follows:
$|\bar{y}_{11.} - \bar{y}_{21.}| = |9.12 - 8.00| = 1.12$
$|\bar{y}_{11.} - \bar{y}_{31.}| = |9.12 - 8.52| = 0.6$
and $|\bar{y}_{21.} - \bar{y}_{31.}| = |8.00 - 8.52| = 0.52$

For factor B fixed at $j = 2$ (Kiswahili), the absolute differences in mean performance were as follows:

$|\bar{y}_{12.} - \bar{y}_{22.}| = |9.96 - 8.28| = 1.68*$

$|\bar{y}_{12.} - \bar{y}_{32.}| = |9.96 - 8.96| = 1.00$

and

$|\bar{y}_{22.} - \bar{y}_{32.}| = |8.28 - 8.96| = 0.68$

And finally when factor B was fixed at $j = 3$ (mathematics), the absolute differences in mean performance were as follows:

$|\bar{y}_{13.} - \bar{y}_{23.}| = |9.08 - 5.12| = 3.96*$

$|\bar{y}_{13.} - \bar{y}_{33.}| = |9.08 - 5.72| = 3.36*$

and

$|\bar{y}_{23.} - \bar{y}_{33.}| = |5.12 - 5.72| = 0.60$

The starred values indicate pairs of means that were significantly different. These were;

(1) the mean performance in Kiswahili between Boys schools and Girls schools,

(2) the mean performance in Mathematics between Boys schools and Girls Schools and

(3) the mean performance in Mathematics between Boys Schools and Mixed Schools.

There was no evidence the rest of the pairs of mean performances were statistically different

4.3 Model for Performance in Compulsory Subjects

The effects model $y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$ is assumed, where $i = 1, 2, 3$, $j = 1, 2, 3$ and $k = 1, 2, ...25$. Since the design is two-factor factorial with fixed effects, the terms are defined as:

μ= the overall mean performance

α_i and β_j are the fixed treatment effects due to school type and subject respectively and are defined as the deviations from the overall mean performance μ.

Hence $\sum_{i=1}^{3} \alpha_i = 0$ and $\sum_{j=1}^{3} \beta_j = 0$.

Also $(\alpha\beta)_{ij}$ is the fixed interaction effect of school type and subject in the $(ij)^{th}$ cell and is defined in such a manner that $\sum_{i=1}^{3}(\alpha\beta)_{ij} = \sum_{j=1}^{3}(\alpha\beta)_{ij} = 0$ and ε_{ijk} is the measure of the deviation of the observed value of performance y_{ijk} in the $(ij)^{th}$ cell from the cell mean performance μ_{ij} .

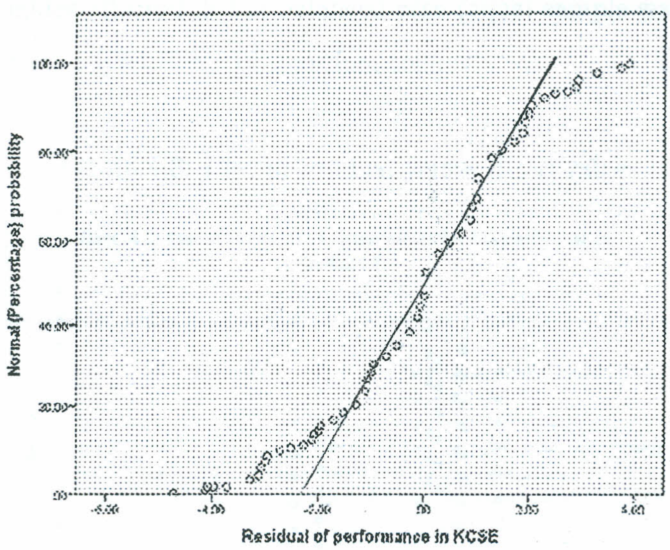
4.3.1 Model Adequacy Checking for Performance in Compulsory Subjects in K.C.S.E

Table 5.5 below shows residuals for performance in compulsory subjects in K.C.S.E. The first column deals with the observational order for the entire sample, the second deals with school type, the third deals with subject, the forth deals with observational order per cell, the fifth deals with the actual observation, the sixth deals with the residual and the last deals with the fitted (predicted) value.

4.3.2 Normality Assumption Checking for Performance in Compulsory Subjects

The normal probability plot of the residuals (in table5.5) is as shown in figure 4.2 below. Note that in drawing the straight line through the points, greater attention is paid to points between the 25^{th} and 75^{th} percentile points rather than points outside this range.

Figure 4.2: Normal (percentage) probability plot of residual of performance in KCSE



Visual examination of this plot reveals about six extreme residuals (-4.72,-4.12, -4.08, -3.96, 3.72 and 3.88) which might be troublesome in the analysis. However, taking the standardized value of the biggest residual (in magnitude) among the six i.e -4.72 gives

$d_{3319} = \frac{e_{3319}}{\sqrt{MS_E}}$

$= \frac{-4.72}{\sqrt{3.229}}$

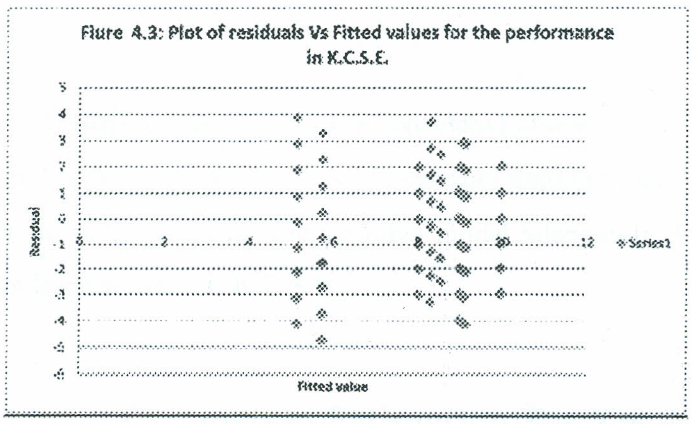
$= -2.6267$

Since this standardized (d_{3319}) value is less than 3 in magnitude, its effect on the $ANOVA$ is negligible and so is the case for the effects of the other five extreme residuals. This shows that the normality assumption is satisfied by the model, hence the adequacy of the model as far as the normality condition is concerned.

4.3.3 Constant Variance Checking For Performance in Compulsory Subjects

Figure 4.3 below is a plot of residuals against the fitted values of y_{iik} . The points in this plot are uniformly spread out in a structure less pattern about the residual mean (zero) line. This is an indication of model adequacy as far as constancy of variance is concerned. In any case if the assumption of the constancy of variance is violated, the F-test is only slightly affected in the balanced case (equal sample sizes in all treatments) for the fixed effects model. This is according to[9].

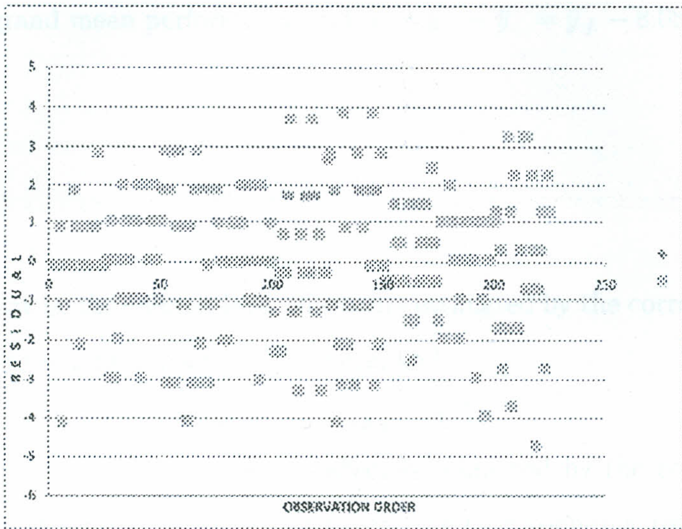
Figure 4.3: Plot of residual versus fitted values for the performance in KCSE



4.3.4 Independence Assumption Checking

Figure 4.4 below is a plot of the residuals in time sequence (observational order). Clearly the points on the plot are uniformly spread out in a structureless pattern about the residuals mean (zero) line. Thus there is no reason to suspect any violation of the independence assumption, hence the adequacy of the model.

Figure 4.4: Plot of residual versus Observational order for the performance in KCSE



Clearly the residuals plots considered above do not indicate any violation of the normality assumption, the constant variance assumption and independence assumption. This is an indication that the assumed model adequately describes the performance in compulsory subject in K.C.S.E.

4.3.5 Parameters Estimation For The Model Of Performance in Compulsory Subjects

Given the fixed effects model for the performance in compulsory subjects in KCSE as $y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$ and from equations 3.34,3.35 ,3.36 and 3.37, the parameters μ, α_i, β_j and $(\alpha\beta)_{ij}$ are respectively estimated as follows; $\hat{\mu} = \bar{y}_{...} = 8.08$

that is, the overall population mean is estimated by the grand mean performance.

$\hat{\alpha}_i = \bar{y}_{i..} - \bar{y}_{...} = \bar{y}_{i..} - 8.08,$

That is, the row level effects are estimated by the corresponding row level mean minus the grand mean performance and $\hat{\beta}_j = \bar{y}_{.j.} - \bar{y}_{...} = \bar{y}_{.j.} - 8.08$,

that is, the column level effects are estimated by the corresponding column level mean minus the grand mean performance; and

$\hat{(\alpha\beta)}_{ij} = \bar{y}_{ij.} - \bar{y}_{...} - (\bar{y}_{i..} - \bar{y}_{...}) - (\bar{y}_{.j.} - \bar{y}_{...})$

That is, the $(ij)^{th}$ interaction effect is estimated by the corresponding $(ij)^{th}$ cell mean minus the grand mean performance, the corresponding row level effect and the corresponding column level effect. This simplifies as follows;

$\hat{(\alpha\beta)}_{ij} = \bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...} = \bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + 8.08$

Also from equation 3.38

$\varepsilon_{ijk} = y_{ijk} - \bar{y}_{ij.}$

That is, the error due to unexplained source in the recording of an observation in the data for the performance in compulsory subjects in KCSE as displayed in table 4.1 is the value of the observation minus the corresponding cell mean performance.

Table 4.3: Summary of the cell means,level means and grand mean and Estimates of (α_i, β_j) and $(\alpha\beta)_{ij}$

	Subjects			
School type	English	Kiswahili	Maths	Mean
Boys school	$\bar{y}_{11.} = 9.12$ $(\hat{\alpha\beta})_{11} = -0.74$	$\bar{y}_{12.} = 9.96$ $(\hat{\alpha\beta})_{12} = -0.42$	$\bar{y}_{13.} = 9.08$ $(\hat{\alpha\beta})_{13} = 1.13$	$\bar{y}_{1..} = 9.39$ $\hat{\alpha}_1 = 1.31$
Girls school	$\bar{y}_{21.} = 8.00$ $(\hat{\alpha\beta})_{21} = 0.40$	$\bar{y}_{22.} = 8.28$ $(\hat{\alpha\beta})_{22} = 0.16$	$\bar{y}_{23.} = 5.12$ $(\hat{\alpha\beta})_{23} = -1.57$	$\bar{y}_{2..} = 7.13$ $\hat{\alpha}_2 = -0.95$
Mixed school	$\bar{y}_{31.} = 8.52$ $(\hat{\alpha\beta})_{31} = -0.32$	$\bar{y}_{32.} = 8.96$ $(\hat{\alpha\beta})_{32} = 0.24$	$\bar{y}_{33.} = 5.72$ $(\hat{\alpha\beta})_{33} = -0.57$	$\bar{y}_{3..} = 7.73$ $\hat{\alpha}_3 = -0.35$
Mean	$\bar{y}_{.1.} = 8.55$ $\hat{\beta}_1 = 0.47$	$\bar{y}_{.2.} = 9.07$ $\hat{\beta}_2 = 0.99$	$\bar{y}_{.3.} = 6.64$ $\hat{\beta}_3 = -1.44$	$\bar{y}_{...} = 8.08$

From table 4.3 above

$\hat{\mu} = \bar{y}_{...} = 8.08$

if $\bar{y}_{i..} = \bar{y}_{1..} = 9.39$ then,

$\hat{\alpha}_1 = \bar{y}_{1..} - \bar{y}_{...} = 9.39 - 8.08 = 1.31$

That is, the effect of school type $1(i = 1) \Longrightarrow$boys schools) on the performance in compulsory subjects in K.C.S.E is 1.31. Also $\bar{y}_{.j.} = \bar{y}_{.1.} = 8.55$, implies $\beta_1 = \hat{\bar{y}_{.1.}} - \bar{y}_{...} =$

= 8.55 -8.08

= 0.47

That is, the effect of English $(i = 1)$ on the performance in compulsory subjects in K.C.S.E is 0.47. This further implies that: $\hat{(\alpha\beta)}_{ij} = (\hat{\alpha\beta})_{11} =$

$= \bar{y}_{11.} - \bar{y}_{1..} - \bar{y}_{.1.} + \bar{y}_{...}$

$= 9.12 - 9.39 - 8.55 + 8.08$

$= -0.74$

That is, the effect due to interaction between school type1 (i=1 implying boys schools) and subject 1(j=1 implying English) on the performance in compulsory subjects in K.C.S.E is -0.74. From equation 3.38, if $\varepsilon_{iik} = \varepsilon_{111}$, then,

$\varepsilon_{111} = y_{111} - \bar{y}_{11}.$

$= 89.12$

$= -1.12$

That is, the error due to unexplained source in the first value of the observed performance is -1.12. Now to adequately describe an observation like 8 (the first observation) in the data for the performance in compulsory subjects in K.C.S.E as displayed in table 4.1, then $y_{ijk} = y_{111} = \mu + \alpha_1 + \beta_1 + (\alpha\beta)_{11} + \varepsilon_{111}$

$= 8.08 + 1.31 + 0.47 + (-0.74) + (-1.12)$

$= 8$

. Since $y_{111} = 8$ tallies with the first observation in the data shown in table 4.1, it implies that the fixed effects model adequately describes the first observation (8).

4.4 Comparing The Fixed-Effects Model Value Of y_{ijk} with the Observed Value

Table 5.6 below gives a summary of the sum of the parameters of the fixed effects model, leading to the adequate description of the various observations in the data for the performance in compulsory subjects as displayed in table 4.1. In table 4.5, the first column deals with the algebraic representation of the various observations, the second column deals with the estimated grand mean, the third column deals with the effects due to the level of school type, the forth column deals with the effects due to the level of the subject, the firth deals with the effects due to school type subject interaction, the six deals with the effects due to random error, the seventh column gives the result of the sum of the effects (parameters) and the eighth deals with the observed value of the performance.

From the table 5.6 in the appendix 5.2 it is clear that the observed value of y_{ijk} is exactly equal to the value determined using the fixed-effects model $y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$ for $i = 1, 2, 3, j = 1, 2, 3$ and $k = 1, 2, 3 - - - 25$. This is a clear indication that the fixed-effects model $y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$ accurately and adequately describes the observed performance in compulsory subjects.

Chapter 5

DISCUSSION, CONCLUSION AND RECOMMENDATION

5.1 Discussion and Conclusion

The analysis in the previous chapter resulted in a number of findings consistent with the objectives of the study. Both school type and subject were found to have a significant effect at $\alpha = 0.05$. This indicates that the performance depends on both school type and subject. The presence of significant interaction effect is an indication that the overall performance depends not only on school type and subject individually but also jointly. The contention that single-sex schools performance is different from mixed schools performance was only supported in the case of Mathematics where Scheff'e's method showed the mean performance of Boys schools and Girls schools to be significantly different from the mean performance of mixed schools at 5% level of significance. The same method (Scheff'e's method) showed that the mean performance of boys schools and Girl schools was significantly different at $\alpha = 0.05$ from the corresponding mean for mixed schools for both English and Kiswahili. Thus, it can generally be concluded that there were no significant differences (at $\alpha = 0.05$) in mean performances between single-sex school and mixed schools in a majority of the compulsory subject that is, English and Kiswahili. Pairwise comparisons using Tukeys method revealed statistically significant differences in mean performance between boys schools and girls schools in both Mathematics and Kiswahili and between boys schools and mixed schools in Mathematics. The fixed effects model $y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$ for $i = 1, 2, 3, j = 1, 2, 3$ and $k = 1, 2, 3.....25$ was found to adequately and accurately describe the performance in K.C.S.E compulsory subjects. This is due to the perfect equality between the observed value of the performance and the corresponding value as determined using the model.

5.2 Recommendation for further studies

This area has great potential for further studies. Subsequent studies can be conducted involving the other subjects taken at secondary level since the performance of a school is judged from the performance in the collectivity of subjects taken at K.C.S.E. The studies could also involve a category of subjects such as languages, science subjects or humanities/arts. The studies could use data for performance in K.C.S.E over several years instead of just a year or two. Differential performance exists between categories of schools other than those based on gender. Thus some of the futures studies in this area can be dedicated to exploring differential performance between such categories of schools as public schools and private schools, religiously affiliated schools and secular ones, boarding schools and day schools etc. It is recommended that future studies on differential performance focus attention on other institution such as colleges, technical institutes and vocational training institutes. Future studies should involve more counties to find out whether the results can be generalized to all counties across the country or whether they apply to specific counties. Following the successful application of the two-factor factorial design in the study, it is recommended that other factorial designs such as three-factor factorial, 2^k, 3^k etc. factorial designs be used in some of the subsequent studies on differential performance in examinations.

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