Conserved excitation number and U(1)-symmetry operators for the anti-rotating (anti-Jaynes-Cummings) term of the Rabi Hamiltonian

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Introduction The quantum Rabi model describes the dynamics of a quantized electromagnetic field mode interacting with a two-level atom generated by Hamiltonian [1-5]

$$H_R = \frac{1}{2}\hbar\omega \left(\hat{a}^{\dagger}\hat{a} + \hat{a}\hat{a}^{\dagger}\right) + \hbar\omega_0 s_z + \hbar g(\hat{a} + \hat{a}^{\dagger})(s_+ + s_-)$$
(1a)

where ω , \hat{a} , \hat{a}^{\dagger} are the quantized field mode angular frequency, annihilation and creation operators, while ω_0 , s_z , s_+ , s_- are the atomic state transition angular frequency and operators. We have used $\sigma_x = s_- + s_+$ and expressed the free field mode Hamiltonian in the symmetrized normal and anti-normal order form $\frac{1}{2}\hbar\omega(\hat{a}^{\dagger}\hat{a}+\hat{a}\hat{a}^{\dagger})$ for reasons which will become clear below.

Collecting the normal and anti-normal order terms in equation (1a), we express the Rabi Hamiltonian in the symmetrized form

$$H_R = \frac{1}{2}(H + \overline{H}) \tag{1b}$$

where we have identified the normal order rotating component as the Jaynes-Cummings Hamiltonian H obtained as

$$H = \hbar \left(\omega \hat{a}^{\dagger} \hat{a} + \omega_0 s_z + 2g(\hat{a}s_+ + \hat{a}^{\dagger}s_-) \right)$$

$$\tag{1c}$$

and the anti-normal order anti-rotating component as the anti-Jaynes-Cummings Hamiltonian \overline{H} obtained as

$$\overline{H} = \hbar \left(\omega \hat{a} \hat{a}^{\dagger} + \omega_0 s_z + 2g(\hat{a} s_- + \hat{a}^{\dagger} s_+) \right)$$
(1d)

We observe that the operator ordering principle which distinguishes the rotating (Jaynes-Cummings) and anti-rotating (anti-Jaynes-Cummings) components H, \overline{H} is not arbitrary, but has physical foundation. Noting that an electromagnetic field mode is composed of positive and negative frequency components [6], we provide the physical interpretation that the Jaynes-Cummings interaction represents the coupling of the atomic spin to the rotating positive frequency component of the field mode, such that the algebraic operations which generate the resulting red-sideband state transitions are achieved through normal-operator-ordering, while the anti-Jaynes-Cummings interaction represents the coupling of the atomic spin to the anti-rotating negative frequency component of the field mode, such that the algebraic operations which generate the resulting blue-sideband state transitions are achieved through anti-normal-operator-ordering. We note that blue-sideband effects arising from interactions involving negative frequency radiation have been observed in recent experiments [7, 8].

In [2], the Jaynes-Cummings and ant-Jaynes-Cummings Hamiltonians H, \overline{H} have been characterized as the *chiral* and *anti-chiral* components, respectively, of the Rabi Hamiltonian. In this respect, we generalize the models of interaction between a single quantized field mode and a single two-level atom to include the asymmetric (anisotropic) Rabi models [3, 4] by introducing a *rotation-symmetry or chirality parameter r* taking values $-1 \le r \le 1$ to express the Rabi Hamiltonian in equation (1b) in general symmetrization form

$$H_R = \frac{1}{2}((1+r)H + (1-r)\overline{H})$$
 ; $-1 \le r \le 1$ (1e)

such that r = 1, 0, -1 specifies that the Rabi Hamiltonian takes respectively the fully rotating (Jaynes-Cummings), symmetric (isotropic) or fully anti-rotating (anti-Jaynes-Cummings) form, while for all other values $r \neq 1, 0, -1$ the Rabi Hamiltonian is asymmetric (anisotropic).

A major challenge, which has remained an outstanding problem in the Rabi model over the years, is that while the Jaynes-Cummings component has a conserved excitation number operator and is invariant under the corresponding U(1)-symmetry operation [1 , 3], a conserved excitation number and corresponding U(1)-symmetry operators for the anti-Jaynes-Cummings component have never been determined, leading to the general belief that the anti-Jaynes-Cummings interaction violates energy conservation principle. We address the long outstanding problem of excitation number and U(1)-symmetry operators of the anti-Jaynes-Cummings Hamiltonian in this letter.

Excitation number operators It follows from the physical interpretation given above that an excitation number operator in the Jaynes-Cummings interaction should be defined in normal-order form, while an excitation number operator in the anti-Jaynes-Cummings interaction should be defined in anti-normal-order form. Taking this operator ordering principle into account, we add and subtract an atomic spin normal order term $\hbar\omega s_+s_-$ in equation (1c) and anti-normal order term $\hbar\omega s_-s_+$ in equation (1d), then reorganize, noting $s_+s_-=\frac{1}{2}+s_z$, $s_-s_+=\frac{1}{2}-s_z$, to obtain the Jaynes-Cummings Hamiltonian in the form

$$H = \hbar\omega(\hat{a}^{\dagger}\hat{a} + s_{+}s_{-}) + 2\hbar g(\alpha s_{z} + \hat{a}s_{+} + \hat{a}^{\dagger}s_{-}) - \frac{1}{2}\hbar\omega \qquad ; \qquad \alpha = \frac{\omega_{0} - \omega}{2q}$$
 (2a)

and the anti-Jaynes-Cummings Hamiltonian in the form

$$\overline{H} = \hbar\omega(\hat{a}\hat{a}^{\dagger} + s_{-}s_{+}) + 2\hbar g(\overline{\alpha}s_{z} + \hat{a}s_{-} + \hat{a}^{\dagger}s_{+}) - \frac{1}{2}\hbar\omega \qquad ; \qquad \overline{\alpha} = \frac{\omega_{0} + \omega}{2g}$$
 (2b)

where we factored out 2g and introduced respective dimensionless frequency-detuning parameters α , $\overline{\alpha}$ defined as indicated.

In the Jaynes-Cummings Hamiltonian H, we identify the normally-ordered Jaynes-Cummings excitation number operator \hat{N} , while in the anti-Jaynes-Cummings Hamiltonian \overline{H} , we identify the anti-normally ordered anti-Jaynes-Cummings excitation number operator \hat{N} , respectively defined by

$$\hat{N} = \hat{a}^{\dagger} \hat{a} + s_{+} s_{-} \qquad ; \qquad \hat{\overline{N}} = \hat{a} \hat{a}^{\dagger} + s_{-} s_{+} \tag{2c}$$

which we introduce in equations (2a), (2b) as appropriate to express the Jaynes-Cummings and anti-Jaynes-Cummings Hamiltonians in the form

$$H = \hbar\omega\hat{N} + 2\hbar g(\alpha s_z + \hat{a}s_+ + \hat{a}^{\dagger}s_-) - \frac{1}{2}\hbar\omega \qquad ; \qquad \overline{H} = \hbar\omega\hat{\overline{N}} + 2\hbar g(\overline{\alpha}s_z + \hat{a}s_- + \hat{a}^{\dagger}s_+) - \frac{1}{2}\hbar\omega \quad (2d)$$

We observe that the Jaynes-Cummings excitation number operator $\hat{N} = \hat{a}^{\dagger}\hat{a} + s_{+}s_{-}$ is a standard conserved operator in the dynamics generated by the rotating component of the Rabi Hamiltonian, while the anti-Jaynes-Cummings excitation number operator $\hat{N} = \hat{a}\hat{a}^{\dagger} + s_{-}s_{+}$, which we establish here as a conserved operator in the dynamics generated by the anti-rotating component of the Rabi Hamiltonian, is a new operator discovered and presented for the first time in the present letter. The discovery of the anti-Jaynes-Cummings excitation number operator, proof of its conservation and specification of the corresponding U(1) and parity symmetry operators in the dynamics generated by the anti-Jaynes-Cummings Hamiltonian are the main results of this letter.

Proof of conservation : state transition operators Using standard atomic spin and field mode operator algebraic relations

$$[s_{+}, s_{-}] = 2s_{z} ; [s_{z}, s_{-}] = -s_{-} ; [s_{z}, s_{+}] = s_{+} ; s_{+}s_{-} = \frac{1}{2} + s_{z} ; s_{-}s_{+} = \frac{1}{2} - s_{z}$$

$$[s_{+}s_{-}, s_{+}] = s_{+} ; [s_{-}s_{+}, s_{+}] = -s_{+} ; [s_{+}s_{-}, s_{-}] = -s_{-} ; [s_{-}s_{+}, s_{-}] = s_{-}$$

$$\hat{a}\hat{a}^{\dagger} = \hat{a}^{\dagger}\hat{a} + 1 ; [\hat{a}^{\dagger}\hat{a}, \hat{a}] = -\hat{a} ; [\hat{a}^{\dagger}\hat{a}, \hat{a}^{\dagger}] = \hat{a}^{\dagger}$$

$$(3a)$$

we easily prove that the excitation number operators \hat{N} , \overline{N} in equation (2c) commute with the respective Jaynes-Cummings and anti-Jaynes-Cummings Hamiltonians H, \overline{H} in equation (2d) according to

$$[\hat{N}, H] = 0 \quad ; \quad [\hat{\overline{N}}, \overline{H}] = 0$$
 (3b)

which proves the standard dynamical property that the excitation number operator $\hat{N}=\hat{a}^{\dagger}\hat{a}+s_{+}s_{-}$ is conserved in the dynamics generated by the Jaynes-Cummings Hamiltonian H and the new dynamical

property that the excitation number operator $\hat{N} = \hat{a}\hat{a}^{\dagger} + s_{-}s_{+}$ is conserved in the dynamics generated by the anti-Jaynes-Cummings Hamiltonian \overline{H} .

To make the proof even more transparent, we introduce two new conserved dynamical operators, namely, Jaynes-Cummings state transition operator \hat{A} and anti-Jaynes-Cummings state transition operator \hat{A} , respectively defined by

$$\hat{A} = \alpha s_z + \hat{a}s_+ + \hat{a}^{\dagger}s_- \qquad ; \qquad \hat{\overline{A}} = \overline{\alpha}s_z + \hat{a}s_- + \hat{a}^{\dagger}s_+ \tag{3c}$$

which on squaring and applying standard atomic spin and field mode operator algebraic relations

$$s_z^2 = \frac{1}{4}\;; \quad s_-^2 = s_+^2 = 0\;; \quad s_+ s_- + s_- s_+ = 1\;; \quad s_z s_+ + s_+ s_z = 0\;; \quad s_z s_- + s_- s_z = 0\;; \quad \hat{a} \hat{a}^\dagger = \hat{a}^\dagger \hat{a} + 1 \ \, (3d)$$

provide the respective Jaynes-Cummings and anti-Jaynes-Cummings excitation number operators \hat{N} , $\overline{\hat{N}}$ defined in equation (2c) in the form

$$\hat{A}^2 = \hat{a}^{\dagger} \hat{a} + s_+ s_- + \frac{1}{4} \alpha^2 = \hat{N} + \frac{1}{4} \alpha^2 \qquad ; \qquad \hat{\overline{A}}^2 = \hat{a} \hat{a}^{\dagger} + s_- s_+ + \frac{1}{4} \overline{\alpha}^2 - 1 = \hat{\overline{N}} + \frac{1}{4} \overline{\alpha}^2 - 1 \qquad (3e)$$

Substituting \hat{A} , $\hat{\overline{A}}$ from equation (3c) and $\hat{N} = \hat{A}^2 - \frac{1}{4}\alpha^2$, $\hat{\overline{N}} = \hat{\overline{A}}^2 - \frac{1}{4}\overline{\alpha}^2 + 1$ from equation (3e) into equation (2d), we express the Jaynes-Cummings and anti-Jaynes-Cummings Hamiltonians in terms of the respective state transition operators in the form

$$H = \hbar (\ \omega \hat{A}^2 + 2g\hat{A}) - \frac{1}{4}\hbar\omega\alpha^2 - \frac{1}{2}\hbar\omega \qquad ; \qquad \overline{H} = \hbar (\ \omega \overline{\hat{A}}^2 + 2g\hat{\overline{A}}) - \frac{1}{4}\hbar\omega\overline{\alpha}^2 + \frac{1}{2}\hbar\omega \qquad (3f)$$

Using equations (3e) and (3f) easily confirms the commutation relations in equation (3b). In addition, it is easy to establish that the Jaynes-Cummings excitation number operator is not conserved in the dynamics generated by the anti-Jaynes-Cummings Hamiltonian \overline{H} and likewise, the anti-Jaynes-Cummings excitation number operator is not conserved in the dynamics generated by the Jaynes-Cummings Hamiltonian H according to the commutation relations

$$[\hat{N}, \overline{H}] \neq 0$$
 ; $[\hat{\overline{N}}, H] \neq 0$ (3g)

Similarly, the state transition operators \hat{A} , $\hat{\overline{A}}$ are conserved in the dynamics generated by the respective Hamiltonians H, \overline{H} , but not in the dynamics generated by the other component Hamiltonian according to the commutation relations

$$[\hat{A}, H] = 0 \quad ; \quad [\hat{A}, \overline{H}] \neq 0 \quad ; \quad [\hat{\overline{A}}, \overline{H}] = 0 \quad ; \quad [\hat{\overline{A}}, H] \neq 0 \quad (3h)$$

We have thus proved the desired conservation of the anti-Jaynes-Cummings excitation number operator and the state transition operators. We have established in another paper [9] that the Jaynes-Cummings state transition operator \hat{A} generates red-sideband transitions between polariton qubit states arising in the rotating Jaynes-Cummings interaction $2\hbar g(\hat{a}^{\dagger}s_{-}+\hat{a}s_{+})$, while the anti-Jaynes-Cummings state transition operator \hat{A} generates blue-sideband transitions between anti-polariton qubit states arising in the anti-rotating anti-Jaynes-Cummings interaction $2\hbar g(\hat{a}s_{+}+\hat{a}^{\dagger}s_{+})$. Eigenvectors and eigenvalues of the respective Jaynes-Cummings and anti-Jaynes-Cummings Hamiltonians H, H in equation (3f), interpreted as polariton and anti-polariton qubit Hamiltonians, have been determined easily in [9]. The coupling of the atomic spin to the anti-rotating negative frequency component of the field mode leading to blue-sideband transitions accounts for the excitation number and energy conservation in the anti-Jaynes-Cummings interaction.

U(1)-symmetry operators The Jaynes-Cummings excitation number operator $\hat{N} = \hat{a}^{\dagger}\hat{a} + s_{+}s_{-}$ generates a free time evolution operator $U_{0}(t)$ obtained as

$$U_0(t) = e^{-i\omega t \hat{N}} \qquad \Rightarrow \qquad U_0^{\dagger}(t) = e^{i\omega \hat{N}t}$$
 (4a)

which provides field mode and atomic spin operator time evolution in the form

$$U_0^{\dagger}(t)\hat{a}U_0(t) = e^{-i\omega t}\hat{a} \; ; \quad U_0^{\dagger}(t)\hat{a}^{\dagger}U_0(t) = e^{i\omega t}\hat{a}^{\dagger} \; ; \quad U_0^{\dagger}(t)s_-U_0(t) = e^{-i\omega t}s_- \; ; \quad U_0^{\dagger}(t)s_+U_0(t) = e^{i\omega t}s_+ \; (4b)$$

The operator $U_0(t)$ thus generates symmetry operations on the Jaynes-Cummings and anti-Jaynes-Cummings Hamiltonians in equation (2d) in the form

$$U_0^{\dagger}(t)HU_0(t) = H \qquad ; \qquad U_0^{\dagger}(t)\overline{H}U_0(t) = \hbar\omega\hat{\overline{N}} + 2\hbar g(\overline{\alpha}s_z + e^{-2i\omega t}\hat{a}s_- + e^{2i\omega t}\hat{a}^{\dagger}s_+) - \frac{1}{2}\hbar\omega \qquad (4c)$$

which shows that the Jaynes-Cummings excitation number operator generated free time evolution operator $U_0(t) = e^{-i\omega t\hat{N}}$ is a U(1)-symmetry operator of the Jaynes-Cummings Hamiltonian H, but not a symmetry operator of the anti-Jaynes-Cummings Hamiltonian \overline{H} .

On the other hand, the anti-Jaynes-Cummings excitation number operator $\hat{N} = \hat{a}\hat{a}^{\dagger} + s_{-}s_{+}$ generates a free time evolution operator $\overline{U}_{0}(t)$ obtained as

$$\overline{U}_0(t) = e^{-i\omega t \hat{N}} \qquad \Rightarrow \qquad \overline{U}_0^{\dagger}(t) = e^{i\omega \hat{N}t} \tag{4d}$$

which provides field mode and atomic spin operator time evolution in the form

$$\overline{U}_0^{\dagger}(t)\hat{a}\overline{U}_0(t) = e^{-i\omega t}\hat{a} \; ; \quad \overline{U}_0^{\dagger}(t)\hat{a}^{\dagger}\overline{U}_0(t) = e^{i\omega t}\hat{a}^{\dagger} \; ; \quad \overline{U}_0^{\dagger}(t)s_{-}\overline{U}_0(t) = e^{i\omega t}s_{-} \; ; \quad \overline{U}_0^{\dagger}(t)s_{+}\overline{U}_0(t) = e^{-i\omega t}s_{+}$$

$$(4e)$$

The operator $\overline{U}_0(t)$ thus generates symmetry operations on the Jaynes-Cummings and anti-Jaynes-Cummings Hamiltonians H, \overline{H} in equation (2d) in the form

$$\overline{U}_0^{\dagger}(t)\overline{H}\overline{U}_0(t) = \overline{H} \qquad ; \qquad \overline{U}_0^{\dagger}(t)H\overline{U}_0(t) = \hbar\omega\hat{N} + 2\hbar g(\alpha s_z + e^{-2i\omega t}\hat{a}s_+ + e^{2i\omega t}\hat{a}^{\dagger}s_-) - \frac{1}{2}\hbar\omega \qquad (4f)$$

which shows that the anti-Jaynes-Cummings excitation number operator generated free time evolution operator $\overline{U}_0(t) = e^{-i\omega t \overline{N}}$ is a U(1)-symmetry operator of the anti-Jaynes-Cummings Hamiltonian \overline{H} , but not a symmetry operator of the Jaynes-Cummings Hamiltonian H.

Parity-symmetry operator It follows from equations (4c) and (4f) that we can determine a common symmetry operator of both Jaynes-Cummings and anti-Jaynes-Cummings Hamiltonians H, \overline{H} by imposing the free evolution symmetry condition

$$e^{-2i\omega t} = e^{2i\omega t} = 1 \quad \Rightarrow \quad 2\omega t = 2n\pi \quad ; \quad \omega t = n\pi \quad ; \quad n = 1, 2, 3, \dots$$
 (5a)

where n=0 defines the identity operator. Substituting $\omega t=n\pi$ into equations (4a), (4d), we obtain the common Jaynes-Cummings and anti-Jaynes-Cummings symmetry operator $\hat{\Pi}_n(\pi)$ in the form

$$\hat{\Pi}_n(\pi) = U_0(n\pi) = e^{-in\pi\hat{N}} = \overline{U}_0(n\pi) = e^{-in\pi\hat{N}} ; \quad n = 1, 2, 3, \dots$$
 (5b)

which we express in the form

$$\hat{\Pi}_n(\pi) = (e^{-i\pi\hat{N}})^n = (e^{-i\pi\hat{N}})^n = (\hat{\Pi})^n$$
(5c)

from which we identify the standard Jaynes-Cummins and anti-Jaynes-Cummings parity-symmetry operator $\hat{\Pi}$ defined here by

$$\hat{\Pi} = e^{-i\pi\hat{N}} = e^{-i\pi\hat{\overline{N}}} \tag{5d}$$

Substituting $\hat{N}=\hat{a}^{\dagger}\hat{a}+s_{+}s_{-}$, $\hat{N}=\hat{a}\hat{a}^{\dagger}+s_{-}s_{+}$ and using algebraic relations

$$\hat{a}\hat{a}^{\dagger} = \hat{a}^{\dagger}\hat{a} + 1 \quad ; \qquad s_{-}s_{+} = s_{+}s_{-} - 2s_{z} \quad ; \qquad \hat{\overline{N}} = \hat{N} + 2s_{-}s_{+}$$
 (5e)

we obtain

$$e^{-i\pi\hat{N}} = e^{-i\pi\hat{N}}e^{-2i\pi s_- s_+} \quad ; \quad e^{-2i\pi s_- s_+} = I \quad \Rightarrow \quad e^{-i\pi\hat{N}} = e^{-i\pi\hat{N}} \tag{5f}$$

which establishes the common Jaynes-Cummings and anti-Jaynes-Cummings parity-symmetry operator relation in equation (5d).

It is easy to establish that the Jaynes-Cummings and anti-Jaynes-Cummings parity-symmetry operator $\hat{\Pi}$ is a symmetry operator of the Rabi Hamiltonian $H_R = \frac{1}{2}(H + \overline{H})$ in equation (1b) according to the symmetry transformation operations

$$\hat{\Pi}^{\dagger} H \hat{\Pi} = H \qquad ; \qquad \hat{\Pi}^{\dagger} \overline{H} \hat{\Pi} = \overline{H} \qquad ; \qquad \hat{\Pi}^{\dagger} H_R \hat{\Pi} = H_R \tag{5g}$$

Finally, we observe that an important dynamical feature emerges from the Jaynes-Cummings-anti-Jaynes-Cummings common parity-symmetry relation in equation (5d). Substituting $\hat{N} = \hat{A}^2 - \frac{1}{4}\alpha^2$, $\hat{\overline{N}} = \hat{\overline{A}}^2 - \frac{1}{4}\overline{\alpha}^2 + 1$ from equation (3e) into equation (5d) and reorganizing, we obtain the common parity-symmetry relation in the form

$$e^{-i\pi\hat{A}^2} = e^{-i\pi\hat{A}^2} e^{i\pi(\frac{1}{4}\overline{\alpha}^2 - \frac{1}{4}\alpha^2 - 1)}$$
 (6a)

which on using $\alpha=\frac{\omega_0-\omega}{2g}$, $\overline{\alpha}=\frac{\omega_0+\omega}{2g}$ from equations (2a) , (2b) to evaluate

$$\frac{1}{4}\overline{\alpha}^2 - \frac{1}{4}\alpha^2 = \frac{\omega_0\omega}{4g^2} = \beta^2 \tag{6b}$$

takes the form

$$e^{-i\pi\hat{A}^2} = e^{-i\pi\hat{A}^2} e^{i\pi(\beta^2 - 1)}$$
 ; $\beta^2 = \frac{\omega_0 \omega}{4g^2}$ (6c)

which suggests that there exists a *critical coupling constant* g_c at which the global phase factor $e^{i\pi(\beta^2-1)}$ equals unity obtained as

$$g = g_c$$
 ; $e^{i\pi(\beta_c^2 - 1)} = 1$: $\beta_c^2 = \frac{\omega_0 \omega}{4g_c^2} = 1$ \Rightarrow $g_c = \frac{1}{2}\sqrt{\omega_0 \omega}$ (6d)

giving common parity-symmetry relation at the critical coupling g_c in the form

$$g = g_c : \quad \alpha_c = \overline{\alpha}_c = \frac{\omega_0 - \omega}{2g_c} \quad ; \quad \beta_c^2 = \frac{\omega_0 \omega}{4g_c^2} \quad ; \quad \hat{\Pi}_c = e^{-i\pi \hat{A}_c^2} = e^{-i\pi \hat{A}_c^2}$$
 (6e)

We identify $g_c = \frac{1}{2}\sqrt{\omega_0\omega}$ to be exactly the critical coupling constant at which the Rabi interaction undergoes quantum phase transition as determined in a recent study [5]. It follows that parity-symmetry breaking may occur at a quantum phase transition. We have presented quantum phase transition phenomena in the Rabi and the more general Dicke models in another paper.

Conclusion We have applied operator-ordering as the fundamental algebraic property to determine the conserved excitation number and U(1)-symmetry operators for the rotating (Jaynes-Cummings) and anti-rotating (anti-Jaynes-Cummings) components of the Rabi Hamiltonian. The specification of the anti-Jaynes-Cummings excitation number operator means that the eigenvalue spectrum of the anti-Jaynes-Cummings Hamiltonian can now be determined explicitly. The Rabi Hamiltonian is thus composed of two algebraically complete Jaynes-Cummings and anti-Jaynes-Cummings components, each specified by its characteristic excitation number, state transitions, U(1)-symmetry and red or blue sideband eigenvalue spectrum. We have determined the parity-symmetry operator as the common symmetry operator for both Jaynes-Cummings and anti-Jaynes-Cummings components, leading to the standard algebraic property that parity operator is a symmetry operator of the Rabi Hamiltonian. The parity-symmetry may break at a critical coupling constant q_c where quantum phase transition occurs.

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References

- [1] D Braak 2011 On the Integrability of the Rabi Model, Phys.Rev.Lett.107, 100401; arXiv:1103.2461 v2 [quant-ph]
- [2] B C da Cunha, M C de Almeida and A R de Queiroz 2016 On the existence of monodromies for the Rabi model, J.Phys.A: Math.Theor.49, 194002
- [3] Q Xie, H Zhong, M T Batchelor and C Lee 2016 The quantum Rabi model: solutions and dynamics, arXiv:1609.00434 v2 [quant-ph]
- [4] Q Xie, et al 2014 Anisotropic Rabi Model, Phys.Rev.X 4, 021046; arXiv:1401.5865v2[quant-ph]
- [5] M J Hwang, R Puebla and M B Plenio 2015 Quantum phase transition and universal dynamics in the Rabi model, Phys.Rev.Lett. 115, 180404; arXiv: 1503.03090 [quant-ph]

- [6] M Born and E Wolf, 1999 Principles of Optics, Cambridge University Press, Cambridge, England
- [7] E Rubino, et al 2012 Negative-Frequency Resonant Radiation, Phys.Rev.Lett. 108, 253901
- [8] J McLenaghan and F Konig 2014 Few-cycle fiber pulse compression and evolution of negative resonant radiation, New J. Phys.. 16, 063017
- [9] J A Omolo 2018 Polariton and anti-polariton qubits in the Rabi model, Preprint-ResearchGate, DOI:10.13140/RG.2.2.11833.67683