Journal of Innovative Technology and Education, Vol. 4, 2017, no. 1, 83 - 96
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/jite.2017.724

Similarity Transformation Analysis of Heat and Mass Transfer Effects on Steady Buoyancy Induced MHD Free Convection Flow Past an Inclined Surface

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Abstract

The present work is devoted to the problem of a steady two dimensional hydromagnetic convective flow of a viscous, incompressible electrically conducting fluid past an inclined semi-infinite plate in the presence of magnetic field with heat and mass transfer (double diffusion). The convective flow starts under the simultaneous action of the buoyancy forces caused by the variations in density due to temperature and species concentration differences. A scaling group of similarity transformations is applied to the partial differential equations describing the problem under consideration, into a boundary value problem of coupled ordinary differential equations, which along with the boundary conditions are solved numerically by using shooting technique together with Runge-Kutta fourth order method and effects of various parameters on the flow fields are investigated and presented graphically.

Keywords: Buoyancy induced, Natural convection, Inclined surface, MHD fluid, shooting technique, Runge-Kutta method, double diffusive, similarity technique

1 Introduction

A fluid is a substance whose constituent particles may continuously change their positions relative to one another when shear force is applied to it. As fluid flows, heat is transferred from one point to another. Heat transfer in fluids is called convection. The phenomenon of natural convection arises in fluids when temperature change causes density variation leading to buoyancy forces acting on the fluid particles. Basically, a free convection flow can be described as a transport process in which fluid motion is caused simply by the interaction of a difference in fluid density with the gravitational force field. Often, the buoyancy stratification is achieved by a temperature field which produces the density difference. Such temperature induced density differences are evidenced in atmospheric and oceanic circulations, in the cooling of electronic components, and in the air currents which arise from a cooling object. In addition, the density difference may also be due to a varying composition or phase of a fluid, as in moist air rising, in ocean circulations due to differences in salinity or in suspended particulate matter, or in a mixture of liquid and vapour in a steam generator or processing device. In such flows, the velocity and temperature fields are completely coupled since the flow arises as a result of buoyancy force, which is induced by temperature gradient between the surface and the fluid.The study of heat transfer is integral part of natural convection flow and belongs to the class of problems in boundary layer theory. A large number of physical phenomena involve natural convection (Jaluria [12]), which are enhanced and driven by internal heat generation.Fluids flowing in engineering devices occur within magnetic field. Fluid flow in the presence of a magnetic field is called hydro magnetic flow and the study of hydro magnetic flows is called Magneto Hydro Dynamics (MHD). When a conductive fluid moves through a magnetic field and an ionized gas is electrically conductive, the fluid may be influenced by the magnetic field. Magnetohydrodynamics free convection heat transfer flow is of considerable interest in the technical field due to its frequent occurrence in industrial technology and geothermal application, liquid metal fluids, MHD power generation systems and boundary layer control in aerodynamics.

In nature there exist flows that are caused not only by temperature differences but also by concentration differences. In industries many transport processes exists in which heat and mass transfer take place simultaneously as a result of combined buoyancy effect of thermal diffusion and mass diffusion through chemical species. A comprehensive literature on various aspects of natural convectional flow and its application coupled heat and mass transfer driven by buoyancy due to temperature and concentration variations in a medium, has several important application in geothermal and geophysical engineering such as dispersion of dissolvent materials in flows, under ground disposal of

nuclear waste and chemical engineering processes. Buoyancy forces resulting from thermal and mass diffusion is of considerable interest in nature and many industrial applications such as geophysics, solidification of binary alloys and drying process.
In similarity solution methods we take advantage of this observation and attempt to define an independent variable so that with a coordinate transformation we will transform the boundary layer equations (which are partial differential equations originally) into ordinary differential equations (ODEs). Similarity solutions are not possible for all flow fields and boundary conditions. However, when a similarity solution is possible, then the solution can be considered exact. Most studies have appeared to consider the single scalar quantities or combined effects with vertical orientations, however, little has been done on the combined effects of buoyancy induced forces and double diffusive convection at an inclined infinite plate surface on the boundary layer flow, which is the impetus of this paper.

2 Related Works

In light of these applications, Umemura and Law [2] developed a generalized formulation for the natural convection boundary layer flow over a flat plate with arbitrary inclination. They found that the flow characteristics depend not only on the extent of inclination but also on the distance from the leading edge. Hossain et al. [8] studied the free convection flow from an isothermal plate inclined at a small angle to the horizontal. Recently, Anghel et al. [6] presented a numerical solution of free convection flow past an inclined surface. Very recently, Chen [3] performed an analysis to study the natural convection flow over a permeable inclined surface with variable wall temperature and concentration. He observed that increasing the angle of inclination decreases the effect of buoyancy force. Moreover, Bataller [10] presented a numerical solution for the combined effects of thermal radiation and convective surface heat transfer on the laminar boundary layer about a flat plate in a uniform stream of fluid (Blasius flow) and about a moving plate in a quiescent ambient fluid (Sakiadis flow). Elbashbeshy [7] studied the heat and mass transfer along a vertical plate under the combined buoyancy effects of thermal and species diffusion, in the presence of magnetic field. Aziz [1] investigated a similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition. Ibrahim et al. [4] investigated the similarity reductions for problems of radiative and magnetic field effects on free convection and mass-transfer flow past a semi-infinite flat plate. They obtained new similarity reductions and found an analytical solution for the uniform magnetic field by using Lie group method. They also presented the numerical results for the non-uniform magnetic field. The importance of similarity transformations and their applications to partial differential equations was studied by Pakdemirli and Yurusoy [9]. They investigated the special group transformations for producing similarity solutions. They also discussed spiral group of transformations. In this article, application of scaling group of transformation has been applied to combined Heat and mass transfer convection (double diffusive convection) effects on steady buoyancy induced flow in an inclined plate in the presence of MHD.

3 Geometry of the Problem

We consider a steady two-dimensional hydromagnetic flow of a viscous incompressible, electrically conducting fluid past a semi-infinite inclined plate with an acute angle γ to the vertical. The flow is assumed to be in the x-direction, which is taken along the semi-infinite inclined plate and y-axis normal to it. A magnetic field of uniform strength B_0 is introduced normal to the direction of the flow. In the analysis, we assume that the magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected in comparison to the applied magnetic field. It is also assumed that all fluid properties are constant except that of the influence of the density variation with temperature and concentration in the body force term. The surface is maintained at a constant temperature T_w, which is higher than the constant free stream temperature T_∞ and the chemical species concentration at the plate surface C_w is greater than the constant concentration C_∞. Then under the usual Boussinesq's and boundary layer approximations, the governing equations (see Ramadan and Chamkha [5] and Sivasankaran et al. [11]) are given by:

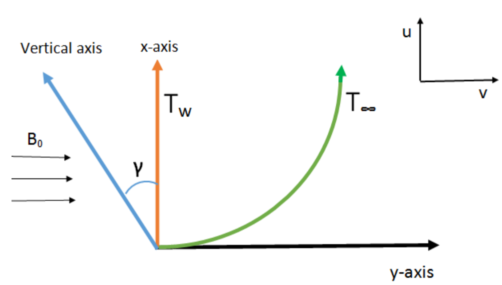

Figure 3.1: Flow Geometry

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta_T(T - T_\infty)\cos\gamma +$$

$$g\beta_C(C - C_\infty)\cos\gamma - \frac{\sigma_c B_0^2}{\rho}u \tag{3.2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3.3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \tag{3.4}$$

The boundary conditions at the plate surface and far into the cold fluid may be written as

$$u(x,0) = 0, \quad v(x,0) = 0, \quad T(x,0) = T_w,$$
$$C(x,0) = C_w \quad \text{at } \mathrm{y} = 0 \tag{3.5}$$

$$u(x,y) \to U_\infty = 0, \;\; T(x,y) \to T_\infty, \;\; C(x,y) \to C_\infty \;\; \text{as}$$
$$y \to \infty \tag{3.6}$$

We now introduce a two-dimensional stream function $\psi(x,y)$ defined by $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$ so that continuity equation is automatically satisfied. In order to obtain a similarity solution of the problem we introduce the following non dimensional variables:

$$\eta = y\sqrt{\frac{U_\infty}{\nu x}}, \;\; \psi(x,y) = \sqrt{\nu U_\infty x}f(\eta), \;\; u = U_\infty f'(\eta),$$

$$v = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}(\eta f'(\eta) - f), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{3.7}$$

where η is a similarity variable, $\theta(\eta)$ and $\phi(\eta)$ are the dimensionless temperature and concentration respectively, U_∞ is the velocity of the fluid far away from the plate. Now substituting equation (3.7) in equations (3.2)- (3.4) we obtain:

$$\frac{\partial u}{\partial x} = \frac{\partial^2\psi}{\partial x\partial y} = \left[\eta_x\eta_y\frac{\partial^2\psi}{\partial\eta^2} + \eta_{xy}\frac{\partial\psi}{\partial\eta}\right]$$

or

$$\frac{\partial u}{\partial x} = -\frac{U_\infty}{2x}(f' + \eta f'')$$

hence

$$u\frac{\partial u}{\partial x} = -\frac{U_\infty^2}{2x}(f'f' + \eta f'f'') \tag{3.8}$$

Also

$$\frac{\partial u}{\partial y} = \frac{\partial^2\psi}{\partial y^2} = U_\infty\sqrt{\frac{U_\infty}{\nu x}}f''$$

hence

$$v\frac{\partial u}{\partial y} = \frac{U_\infty^2}{2x}(\eta f' f'' - ff'') \tag{3.9}$$

Again

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial^3 \psi}{\partial y^3} = \eta_y^3 \frac{\partial^3 \psi}{\partial \eta^3}$$

or

$$\frac{\partial^2 u}{\partial y^2} = \frac{U^2\infty}{\nu x} f'''$$

Thus

$$\nu\frac{\partial^2 u}{\partial y^2} = \frac{U^2\infty}{x} f''' \tag{3.10}$$

Introducing a non-dimensional temperature

$$\theta(x,y) = \frac{T - T_\infty}{T_w - T_\infty}$$

$$\frac{\partial \theta}{\partial x} = \frac{\partial T}{\partial x} = \theta' \eta_x$$

But $\eta_x = -\frac{\eta}{2x}$. From 3.3, we have

$$\frac{\partial T}{\partial x} = (T_w - T_\infty)\theta' \eta_x$$

or

$$\frac{\partial T}{\partial x} = -\frac{\eta}{2x}(T_w - T_\infty)\theta'$$

hence

$$u\frac{\partial T}{\partial x} = -\frac{\eta U_\infty}{2x}(T_w - T_\infty)\theta' f' \tag{3.11}$$

Also

$$\frac{\partial \theta}{\partial y} = \frac{\partial T}{\partial y} = \left(\sqrt{\frac{U_\infty}{vx}}\right)(T_w - T_\infty)\frac{\partial \theta}{\partial \eta}$$

and

$$v\frac{\partial T}{\partial y} = \frac{1}{2}\sqrt{\frac{U_\infty v}{x}}(\eta f' - f)\left[\sqrt{\frac{U_\infty}{vx}}(T_w - T_\infty)\frac{\partial \theta}{\partial \eta}\right]$$

Therefore

$$v\frac{\partial T}{\partial y} = \frac{U_\infty}{2x}(T_w - T_\infty)\theta'(\eta f' - f) \tag{3.12}$$

For the last term in 3.3, we have

$$\frac{\partial^2 T}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial \theta}{\partial y}\right)$$

and

$$\frac{\partial^2\theta}{\partial y^2} = \left(\frac{\partial^2\theta}{\partial \eta^2}\right)\left(\frac{\partial \eta}{\partial y}\right)^2$$

Recall $\eta_y = \sqrt{\frac{U_\infty}{vx}}$, thus $(\eta_y)^2 = \frac{U_\infty}{vx}$,

$$\Rightarrow \frac{\partial^2 T}{\partial y^2} = \frac{U_\infty}{vx}(T_w - T_\infty)\theta''$$

hence

$$\alpha\frac{\partial^2 T}{\partial y^2} = \frac{\alpha U_\infty}{vx}(T_w - T_\infty)\theta'' \tag{3.13}$$

Finally for equation 3.4, we have

$$\frac{\partial C}{\partial x} = (C_w - C_\infty)\eta_x\phi'$$

or

$$\frac{\partial C}{\partial x} = -\frac{\eta}{2x}(C_w - C_\infty)\phi'$$

hence

$$u\frac{\partial C}{\partial x} = -\frac{\eta U_\infty}{2x}(C_w - C_\infty)\phi' f' \tag{3.14}$$

Also

$$\frac{\partial C}{\partial y} = (C_w - C_\infty)\eta_y\frac{\partial \phi}{\partial \eta} = (C_w - C_\infty)\sqrt{\frac{U_\infty}{vx}}\phi'$$

and

$$v\frac{\partial C}{\partial y} = \frac{U_\infty}{2x}(C_w - C_\infty)(\eta f' - f)\phi' \tag{3.15}$$

For the last term in this equation we obtain

$$\frac{\partial^2 C}{\partial y^2} = \eta_{yy}(C_w - C_\infty)\phi' + \eta_y^2(C_w - C_\infty)\phi''$$

$$= \frac{U_\infty}{vx}(C_w - C_\infty)\phi''$$

hence

$$D_m\frac{\partial^2 C}{\partial y^2} = D_m\frac{U_\infty}{vx}(C_w - C_\infty)\phi'' \tag{3.16}$$

Substituting these expressions numbered
(3.8-3.16) into momentum, energy and concentration equations, we find three non-linear ordinary differential equations:

$$f''' + \frac{1}{2}ff'' + \frac{1}{2}(f')^2 + \theta Gr\cos\gamma + \phi Gc\cos\gamma$$

$$-Mf' = 0 \tag{3.17}$$

$$\theta'' + \frac{1}{2}Prf\theta' = 0 \tag{3.18}$$

$$\phi'' + \frac{1}{2}Scf\phi' = 0 \tag{3.19}$$

where the prime symbol denotes differentiation with respect to η and

$$Gr = \frac{g\beta_T x(T_w - T_\infty)}{U_\infty^2}, \quad Gc = \frac{g\beta_C x(C_w - C_\infty)}{U_\infty^2}$$

$$Pr = \frac{v}{\alpha}, \quad M = \frac{\sigma_c B_0^2 x}{\rho U_\infty}, \quad Sc = \frac{v}{D_m}, \quad \alpha = \frac{k}{\rho C_p}$$

in which Gr is the local thermal Grashof number, Gc is the solutal or local concentration Grashof number, Sc is the Schimdt number and Pr is the Prandtl number, The corresponding boundary conditions become

$$f = 0, \ f' = 0, \ \theta = 1, \ \phi = 1 \ \text{at} \ \eta = 0$$

$$f' = 0, \ \theta = 0, \ \phi = 0 \ \text{as} \ \eta \to \infty \tag{3.20}$$

4 Numerical Procedure

The similarity transformation converts the non-linear partial differential equations (3.2 - 3.4) into ordinary differential equations. The set of non-linear ordinary differential equations (3.17 - 3.19) with boundary conditions in (3.20) have been solved numerically using shooting method, a technique that converts the boundary value ordinary differential equations into a set of first order initial value ordinary differential equations with Secant iteration. The resulting system is solved by the fourth-order Runge-Kutta method implemented in Mathematica software.

5 Results and Discussion

To analyze the results of the numerical calculations, the dimensionless temperature and concentration distributions for the flow are obtained from equations (3.17 - 3.19) and are displayed in figures below for various governing parameters.Here, we assigned physically realistic numerical values to the embedded parameters in the system in order to gain an insight into the flow structure with respect to temperature and concentration profiles. To be precise, the values of Prandtl number Pr were chosen for air ($Pr = 0.71$), and water ($Pr = 7.0$) .

5.1 Effects of Parameter Variation on Temperature Profiles

Notice from these figures that the fluid temperature attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach the free stream value away from the plate which satisfies the boundary conditions.

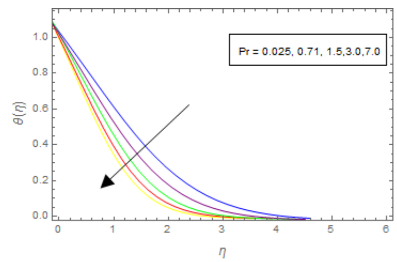

Figure 5.1: Temperature profiles for different values of Pr

From figure 5.1, it is noted that an increase in the Prandtl number results in a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated plate more rapidly than for higher values of Pr. Hence in the case of smaller Prandtl numbers the boundary layer is thicker and the rate of heat transfer is reduced.

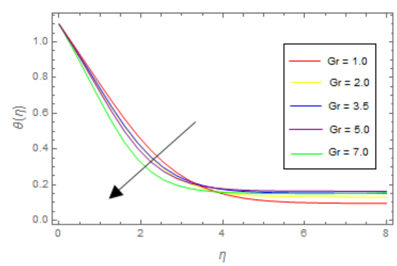

Figure 5.2: Temperature profiles for different values of Gr

Moreover, an increase in the intensity of buoyancy forces (Gr, Gc) causes a decrease in the fluid temperature leading to a decaying thermal boundary layer thickness as displayed in figures 5.2 and 5.3.
Figure 5.4 gives the dimensionless temperature profiles for Schmidt number. The Schmidt number embodies the ratio of the momentum to the mass diffusivity. The Schmidt number therefore quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic and concentration (species) boundary layers. We observed from this figure that the

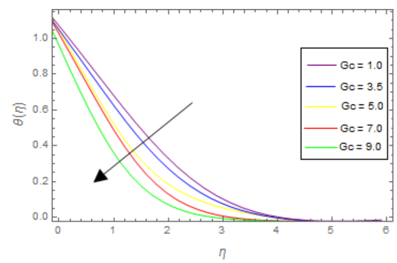

Figure 5.3: Temperature profiles for different values of Gc

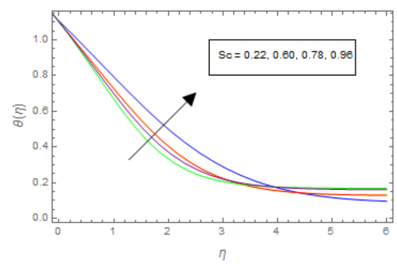

Figure 5.4: Temperature profiles for different values of Sc

temperature profile increases with the increase of the Schmidt number. We also observed that the variation in the thermal boundary layer is very small corresponding to a moderate change in Schmidt number. This shows that the minor increasing effect on the temperature profile is greatly affected by the presence of foreign species.

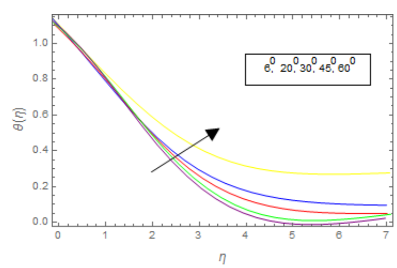

Figure 5.5: Temperature profiles for different values of γ

In figure 5.5, as the angle of inclination increases we observe that both the thermal and concentration boundary layer thickness increase.
For different values of the magnetic field parameter M on the temperature profiles see figure 5.6. It is observed that as the magnetic parameter increases, the temperature also increases i.e. it is worthy to note that the thermal boundary layer thickness increases with an increase in the intensity of the magnetic field parameter, thus the temperature profiles increase with the increase of the magnetic field parameter, which implies that the applied magnetic field tends

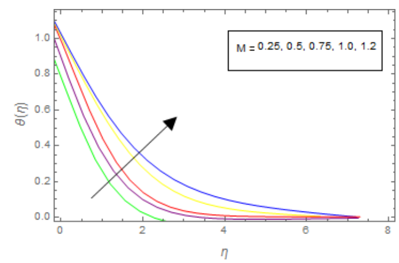

Figure 5.6: Temperature profiles for different values of M

to heat the fluid, and thus reduces the heat transfer from the wall.

5.2 Effects of parameter variation on concentration profiles

Figures 5.7 - 5.11 depict chemical species concentration profiles against span wise coordinate η for varying values physical parameters in the boundary layer. The species concentration is highest at the plate surface and decreases to zero far away from the plate satisfying the boundary condition.

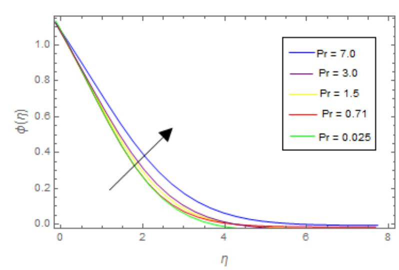

Figure 5.7: Concentration profiles for different values of Pr

The increase in the Prandtl number has an adverse effect on the velocity and temperature profiles of the fluid flow but it has opposite effect on the concentration profile of the fluid along the inclined plate as is clear in figure 5.7. In other words, as we increase the Prandtl number (Pr), concentration profile has increasing trend. i.e. as Pr increases the thickness of the concentration boundary layer increases.

However we observed an increase in the concentration boundary layer when the magnetic parameter was increased as graphically displayed in figure 5.8. Physically, it is true due to the fact that the application of a transverse magnetic field to an electrically conducting fluid gives rise to a body force known as a Lorentz hydromagnetic drag which acts in the tangential direction which retards free convective transfer of fluid mass leaving some molecules stack to

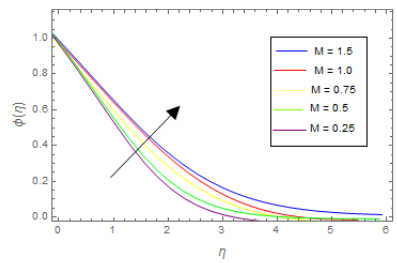

Figure 5.8: Concentration profiles for different values of M

the surface of the plate, resulting in the thickening of the concentration layer.

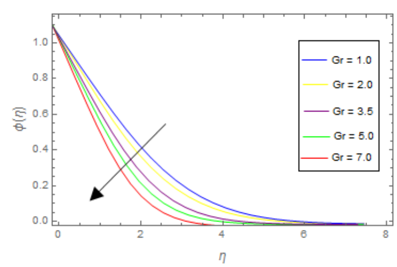

Figure 5.9: Concentration profiles for different values of Gr

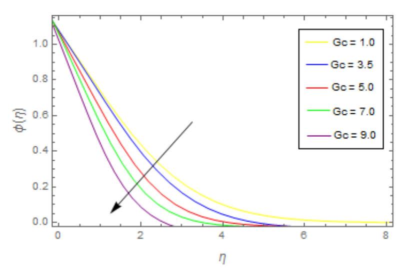

Figure 5.10: Concentration profiles for different Gc

An increase in the values of thermal and solutal Grashof number (Gr, Gc) due to buoyancy forces also causes a decrease in the chemical species concentration leading to a decaying concentration boundary layer thickness. Figure 5.11 represent the effect of aligned angle, on concentration profiles. An increase in thickness of the concentration boundary layer is observed up on increasing the angle of inclination γ. i.e. the concentration of air boundary layer are increased with an increase of γ.

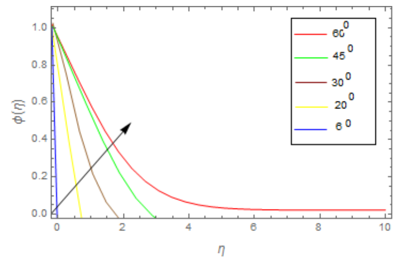

Figure 5.11: Concentration profiles for different values of γ

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Received: March 9, 2017; Published: May 3, 2017